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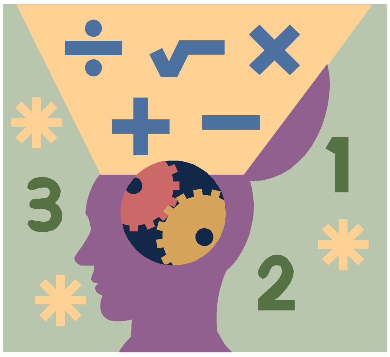
MATHS EXPERIMENTS:
A PRACTICAL RESOURCE
FOR YEAR 9



by

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This book is dedicated
to Jasper and Calypso,
because every good idea
starts somewhere.

Foreword

Without doubt, mathematics has been the most maligned subject in the school curriculum. Many students have seen it as either a form of drudgery or as an unfathomable playground for a few of their classmates.

When mathematics is presented in a manner that is stimulating, enjoyable and not threatening, students develop a real appreciation of the subject. On the one hand, mathematics reveals much that is unique and certain. On the other hand, it emphasises symmetries and other special relationships that continue to unfold and surprise as the years go on. Exercises that enable the student to explore and discover, whilst being entertained, lead to competence and understanding.

This workbook will open up vistas of enjoyable mathematics that will lead to real progress within the subject. It is a valuable resource for problem solving, an important component of essential learning. The author is to be commended for using his experience and insights to assemble this mathematical anthology.

Murray Yaxley, Retired Teacher
Department of Education, Tasmania and UNESCO

Addressing the Year 9 Syllabus

Geometry and Geometrical Patterns

- Draw geometrical figures, angles, triangles, quadrilaterals, 3D objects
- Use geometric techniques to construct angles, lines and 2D figures
- Recognise and name common geometric figures and their parts
- Use geometrical facts, properties and relationships to solve problems
- Recognise symmetry in 2D shapes
- Identify and use the properties of polygons
- Use transformations to draw geometrical patterns
- Identify objects as symmetrical about a plane, line or a point
- Construct a variety of tessellations and simple fractals
- Identify shapes and transformations used in tessellations
- Use compasses, ruler and protractor to construct angles, lines and curves

Measurement

- Use techniques and tools to measure and compare quantities and angles
- Select and use appropriate common units and convert between measures
- Estimate measurements appropriately in various contexts
- Find the perimeters and areas of triangles, circles, squares using formulæ

Trigonometry and Pythagoras' Theorem

- Use Pythagoras' theorem to solve problems
- Use right-angled triangles and trigonometry to solve problems

Algebra

- Use calculation techniques for fractions, decimals, percentages and ratios
- Use written and graphical information to solve problems
- Interpret and use ratios to solve simple problems
- Identify, describe and extend number patterns
- Draw graphs to represent relationships given descriptions or value tables
- Substitute into given formulæ and evaluate the resulting expression
- Solve simple linear equations

Instructions

Before starting any exercise, it is best to read all of the instructions through at least once. Even if you do not understand the task until you actually do it, you should know where you are going before you set off.

The difficulty ratings are: ❁ Simplex, ❁ ❁ Medium, ❁ ❁ ❁ Complex

Glossary of Terms

tri	three (3)
quad	flat four (4 in 2D)
tetra	solid four (4 in 3D)
rect	right, 90 degrees
angle	bend, turn, corner
lateral	side, sides, sided
iso	same
equi	equal
semi	half
dia	across
gonal	corners
meter	measure
radius	stick, rod
circum	round, around

Use the words above to translate the following mathematical phrases:

dia-meter ____

dia-gonal ____

tri-angle ____

tri-lateral ____

iso-lateral ____

equi-lateral ____

tri-equi-lateral ____

semi-circle ____

rect-angle ____

quadri-lateral ____

quad-equi-lateral ____

quad-rect-equi-lateral ____

Table of Contents

Chapter 1

Number Representation

1. Ratios

A ratio is the proportion of one measure to another. As both the measures have the same units, the ratio has no units, as they cancel out in comparison.

1. A fruit juice cordial concentrate may say to mix 1 part cordial to 4 parts water. The ratio of cordial drink to water is written 1:4.

   If making a half litre of cordial (500 mL), the ratio is:

   100 mL of cordial : 400 mL of water. Check: 100 + 400 = 500.

   Write the ratio to make 1 litre of mixed cordial (1000 mL):

   _____mL of cordial : ____mL of water

   Check: _______mL + _______mL = ____________mL

2. An uphill road has a slope (gradient) of 1:5 (1 up for every 5 across).

   Calculate how high you would be if the hill is 200 m across at the base.

   Base of hill is _______m, so half of base of hill = _______m, which is the base of a right angle triangle.

   If height to base ratio is 1:5, then base is 5 times more than the height.

   So, how high is one-fifth of the base, which is _______m ÷ 5 = _______m.

3. The water molecule H_2O or HOH has 2 hydrogen (H) atoms for every oxygen (O) atom. The hydrogen to oxygen ratio is ______:______.

   How many oxygen atoms are needed to make water with 10 hydrogen atoms? 10 hydrogen atoms : ______oxygen atoms, ratio ____ : ____.

4. Glucose sugar has the molecular formula $C_6H_{12}O_6$.

   The ratio of carbon to hydrogen to oxygen is: ____ : ____ : ____.

   Calculate how many glucose molecules can be made if only 18 carbon atoms, 100 hydrogen atoms and 24 oxygen atoms were available by calculating the largest ratio possible, keeping the C : H : O ratio 6:12:6.

   Biggest possible ratio is: ____ : ____ : ____, and because this is _____ times bigger than 6:12:6, it represents ____ glucose molecules.

   Do this by finding the most number of times that each type of atom can fit into what is available for that type of atom, for each of the 3 atoms.

2. Decimals

Decimal is a Latin word meaning a tenth (deci). The decimal point is a full stop half-way between the units and decimals, like 4·5 for four-and-a-half.

The left part represents the whole numbers (4 in the 4·5 example) and the right part represents the part that is less than 1 (0·5 in the 4·5 example).

The 2 decimal digits to the right of the decimal point are the percentage of the part less than 1, as a ratio of 100, and starts with a 0, like 0·5 or 0·75.

The decimal of 3:4 is: 3 ÷ 4 = 0·75

[calculator] 3 ÷ 4 = _______ = 75 %.

This represents that for every 100 parts, the ratio 3:4 means there would be 75, as 3:4 is the same as 75:100 in percentages and decimals.

Use a calculator to write the following as decimals:

1:9	=	1 ÷ 9	=	______________
3:5	=	3 ÷ 5	=	______________
5:5	=	5 ÷ 5	=	______________
10:5	=	10 ÷ 5	=	______________
10:11	=	10 ÷ 11	=	______________
10:12	=	10 ÷ 12	=	______________
12:10	=	12 ÷ 10	=	______________
12:11	=	12 ÷ 11	=	______________
1:100	=	1 ÷ 100	=	______________
14:100	=	14 ÷ 100	=	______________
10:40	=	10 ÷ 40	=	______________
44:88	=	44 ÷ 88	=	______________
22:7	=	22 ÷ 7	=	______________
9.9:3.3	=	9.9 ÷ 3.3	=	______________
1:1.61803398875	=	1 ÷ 1.61803398875	=	______________
1:0.61803398875	=	1 ÷ 0.61803398875	=	______________

3. Percentages

Percentage comes from the Latin words per (through) and cent (hundred). This special ratio is always measured with 100 as being the second number.

1:1 becomes 100:100 or 100%

1:2 becomes 50:100 or 50%

[calculator] 1 ÷ 2 = _____ x 100 = _____%. Try it.

Use a calculator to find the following percentages by dividing the first number of the ratio by the second number of the ratio, and then multiplying the result by 100. Round the answer up to 2 decimal places.

1:3	=	1	÷	3	=	0·33	(x 100) =	33 %.
1:4	=	___	÷	_______	=	_______	(x 100) =	_______ %.
1:5	=	___	÷	_______	=	_______	(x 100) =	_______ %.
1:6	=	___	÷	_______	=	_______	(x 100) =	_______ %.
2:6	=	___	÷	_______	=	_______	(x 100) =	_______ %.
2:8	=	___	÷	_______	=	_______	(x 100) =	_______ %.
2:10	=	___	÷	_______	=	_______	(x 100) =	_______ %.
2:12	=	___	÷	_______	=	_______	(x 100) =	_______ %.
3:9	=	___	÷	_______	=	_______	(x 100) =	_______ %.
3:10	=	___	÷	_______	=	_______	(x 100) =	_______ %.
3:12	=	___	÷	_______	=	_______	(x 100) =	_______ %.
3:24	=	___	÷	_______	=	_______	(x 100) =	_______ %.
4:48	=	___	÷	_______	=	_______	(x 100) =	_______ %.
5:100	=	___	÷	_______	=	_______	(x 100) =	_______ %.
6:108	=	___	÷	_______	=	_______	(x 100) =	_______ %.
7:107	=	___	÷	_______	=	_______	(x 100) =	_______ %.
12:10	=	___	÷	_______	=	_______	(x 100) =	_______ %.

4. Fractions

A fraction is a ratio that is kept as one number divided by another number. Fractions are kept this way because it is more accurate than a decimal number.

The ratio 50:100 is written as a fraction as $\frac{50}{100}$, which means 50 ÷ 100.

Likewise, $\frac{4}{8}$ means 4 ÷ 8, and $\frac{6}{3}$ means 6 ÷ 3.

We also know that $\frac{4}{8}$ is the same ratio as $\frac{2}{4}$, as they are both 1:2.

Use a calculator or multiplication table to convert the following ratios to fractions by multiplying top to top the same as you multiply bottom to bottom to get from one fraction to the next fraction.

Notice that the last fraction is written as a percentage.

$$1:2 = \frac{1}{2} = \frac{\_\_\_}{4} = \frac{\_\_\_}{8} = \frac{8}{\ } = \frac{16}{\ } = \frac{\_\_\_}{50} = \frac{\_\_\_}{100}$$

$$1:3 = \frac{1}{3} = \frac{\_\_\_}{6} = \frac{6}{\ } = \frac{8}{\ } = \frac{12}{\ } = \frac{\_\_\_}{66} = \frac{\_\_\_}{100}$$

$$1:4 = \frac{1}{\ } = \frac{\_\_\_}{8} = \frac{\_\_\_}{16} = \frac{\_\_\_}{40} = \frac{15}{\ } = \frac{\_\_\_}{80} = \frac{\_\_\_}{100}$$

$$1:5 = \frac{1}{5} = \frac{\_\_\_}{10} = \frac{\_\_\_}{25} = \frac{\_\_\_}{60} = \frac{15}{\ } = \frac{\_\_\_}{90} = \frac{\_\_\_}{100}$$

$$3:2 = \frac{3}{\ } = \frac{\_\_\_}{4} = \frac{9}{\ } = \frac{\_\_\_}{20} = \frac{\_\_\_}{40} = \frac{120}{\ } = \frac{\_\_\_}{100}$$

$$12:10 = \frac{\_\_\_}{10} = \frac{\_\_\_}{5} = \frac{\_\_\_}{20} = \frac{36}{\ } = \frac{60}{\ } = \frac{\_\_\_}{70} = \frac{\_\_\_}{100}$$

$$15:10 = \frac{\_\_\_}{10} = \frac{\_\_\_}{12} = \frac{24}{\ } = \frac{\_\_\_}{30} = \frac{\_\_\_}{50} = \frac{90}{\ } = \frac{\_\_\_}{100}$$

5. Roman Numerals ❁ ❁

Up until a few hundred years ago, the western world used the ancient Roman letters to represent numbers, using the following capital letters for numbers:

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000.

To calculate an Arabic numeral (the ones we use) from a Roman numeral, add up the letters from left to right to get the answer. The one exception is when a smaller number is before (to the left of) a bigger number, which means subtract the smaller number from the bigger number.

Then go on adding up the rest of the letters, moving right. For example,

XII	=	12	(10 + 1 + 1)	1 after 1 after 10
XIII	=	13	(10 + 1 + 1 + 1)	1 after 1 after 1 after 10
IX	=	9	(10 - 1)	1 before 10
XIV	=	14	(10 + (5 - 1))	1 before 5 after 10
XV	=	15	(10 + 5)	5 after 10

Translate the following Roman and Arabic numerals:

1	=	______________	37	=	______________
2	=	______________	43	=	______________
3	=	______________	49	=	______________
4	=	______________	54	=	______________
5	=	______________	55	=	______________
6	=	______________	65	=	______________
7	=	______________	72	=	______________
8	=	______________	78	=	______________
9	=	______________	79	=	______________
10	=	______________	80	=	______________
11	=	______________	85	=	______________
12	=	______________	86	=	______________
13	=	______________	90	=	______________
14	=	______________	91	=	______________
15	=	______________	95	=	______________
16	=	______________	96	=	______________
17	=	______________	98	=	______________
18	=	______________	99	=	______________
19	=	______________	101	=	______________
20	=	______________	247	=	______________

CII	=	______________
DIX	=	______________
XXIV	=	______________
MMV	=	______________
LXVIII	=	______________
CMLXXXIV	=	______________
CMXCIX	=	______________
DCLXVI	=	______________
DCCCLXXXVIII	=	______________
MMMMMMMDCCCLXXXVIII	=	______________

6. Binary Numbers

Binary numbers are made up of 2 digits, which are one (1) and zero (0).

1 means 'yes', 'on' or 'full', and 0 means 'no', 'off' or 'empty'.

Calculate the first 11 binary numbers by doubling the previous number, going from right to left. Also fill in the binary powers up to 2^{10}.
('Terms' are whole (natural) numbers.)

10	9	8	7	6	5	4	3	2	1	0	Term
							2^3	2^2	2^1	2^0	2^{Term}
							8	4	2	1	BIN.

Instead of using thousands, hundreds, tens and units, as in decimal Arabic notation, binary uses 8, 4, 2, 1 as the first four numbers, using them to make up the numbers to 1 to 15, as the next binary number is 16.

Calculate the following binary numbers by using a calculator. Start with the number you are going to convert and subtract the biggest binary number that will fit into it, and put a "1" in that binary number column. Then go to the next binary column. If the next number will fit into the remainder, put a "1" in that column as well and subtract it from the remainder. If the next binary number is bigger than the remainder, put a "0" in that column and go on to the next column, until you get down to the last "1" binary number.

	1024	512	256	128	64	32	16	8	4	2	1	Binary
5									1	0	1	
6									1	1	0	
7									1	1	1	
8								1	0	0	0	
9								1	0	0	1	
10								1	0	1	0	
11												
12												
13												
22												
53												
62												
104												
199												
205												
406												
807												
1608												

7. Boolean Logic

A boolean statement that is is either "TRUE" or "FALSE".

Boolean addition is called "AND", and works like this:

If A is TRUE (the sky is blue), and B is TRUE (clouds are white),
then A AND B = TRUE (the sky is blue and clouds are white).
But if A is FALSE, or if B is FALSE, then A AND B = FALSE.

Rule: Both A and B have to be TRUE for A AND B to be TRUE.
All other answers are FALSE.

Boolean subtraction is called "OR", and works like this:

If A is TRUE, and B is TRUE, then A OR B = TRUE
If A is TRUE, and B is FALSE, then A OR B = TRUE
If A is FALSE, and B is TRUE, then A OR B = TRUE
If A is FALSE, and B is FALSE, then A OR B = FALSE.

Rule: Both A and B have to be FALSE for A OR B to be FALSE.
All other answers are TRUE.

Calculate if the following are TRUE or FALSE, if:

A = dogs have 4 legs
B = cats have 4 legs
C = rabbits have wings
D = and they chase mice

A AND B	=	True ______	AND	True _____	=	True _________
A OR B	=	___________	OR	____________	=	_____________
A AND C	=	___________	AND	____________	=	_____________
A OR C	=	___________	OR	____________	=	_____________
A AND D	=	___________	AND	____________	=	_____________
A OR D	=	___________	OR	____________	=	_____________
B AND C	=	___________	AND	____________	=	_____________
B OR C	=	___________	OR	____________	=	_____________
B AND D	=	___________	AND	____________	=	_____________
B OR D	=	___________	OR	____________	=	_____________
C AND D	=	___________	AND	____________	=	_____________
C OR D	=	___________	OR	____________	=	_____________

Chapter 2

Figurative Numbers

8. Triangular Numbers ❀

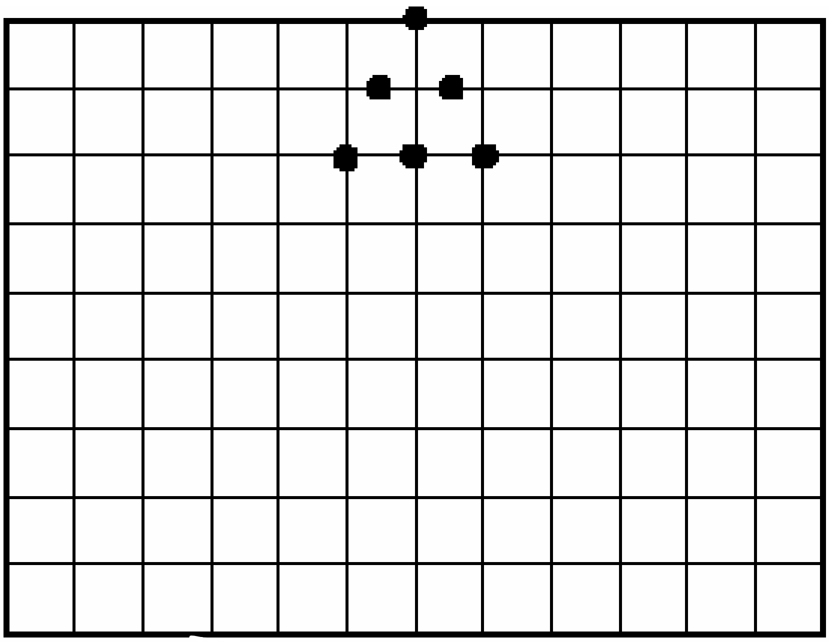
The formula for triangular numbers is:

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + ... and so on.

Triangular numbers are found by adding up the terms of the formula. The terms are the whole (natural) numbers.

Calculate the first 10 triangular numbers by filling in the following table and drawing in the dots to form a big triangle.

The number of dots in each row is added to the total number of dots in the triangle above to give the triangular number for that term.



Term	*Adding*	*Triangle* Number
1	+1 =	1
2	+2 =	3
3	+3 =	
4	+4 =	
5	+5 =	
6	+6 =	
7	+7 =	
8	+8 =	
9	+9 =	
10	+10 =	

Write here a summary of the first 10 triangular numbers:

Term	1	2	3	4	5	6	7	8	9	10
Triangular Number										

9. Square Numbers

The square number formula is:

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + ... and so on.

The square numbers are found by adding up the terms of the formula, which are the odd numbers.

Calculate the first 10 square numbers by filling in the following table and drawing in the dots to form a series of squares from the top left corner.

The odd number of new dots around the old square is added to the total number of dots in the previous square to give the new square number for that term.

Adding Square Number *Term*

1 2 3 4 5 6 7 8 9 10

+ 1 = 1

+ 3 = 4

+ 5 =

+ 7 =

+ 9 =

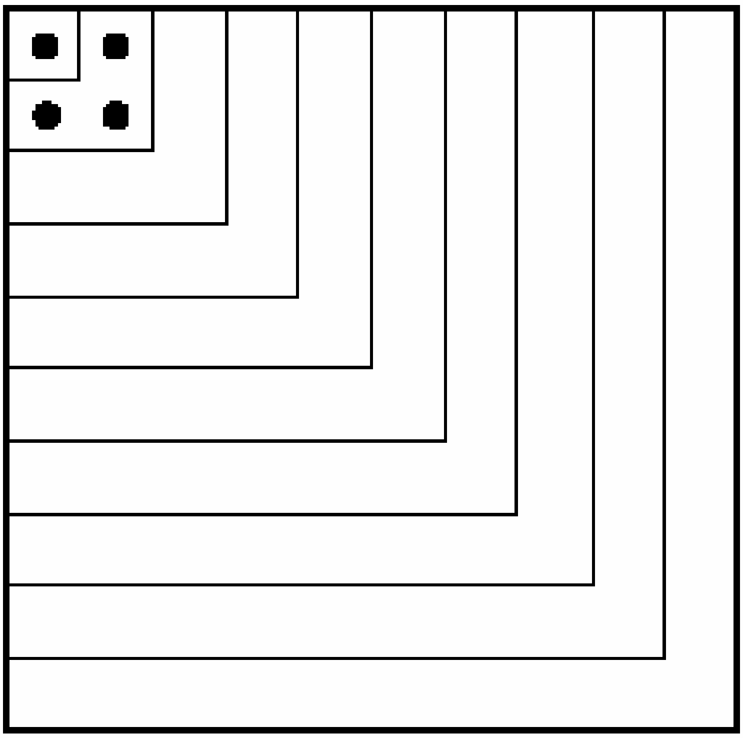
+ 11 =

+ 13 =

+ 15 =

+ 17 =

+ 19 =



Write here a summary of the first 10 square numbers:

Term	1	2	3	4	5	6	7	8	9	10
Square Number										

10. Oblong Numbers

The oblong number formula is:

2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + ... and so on.

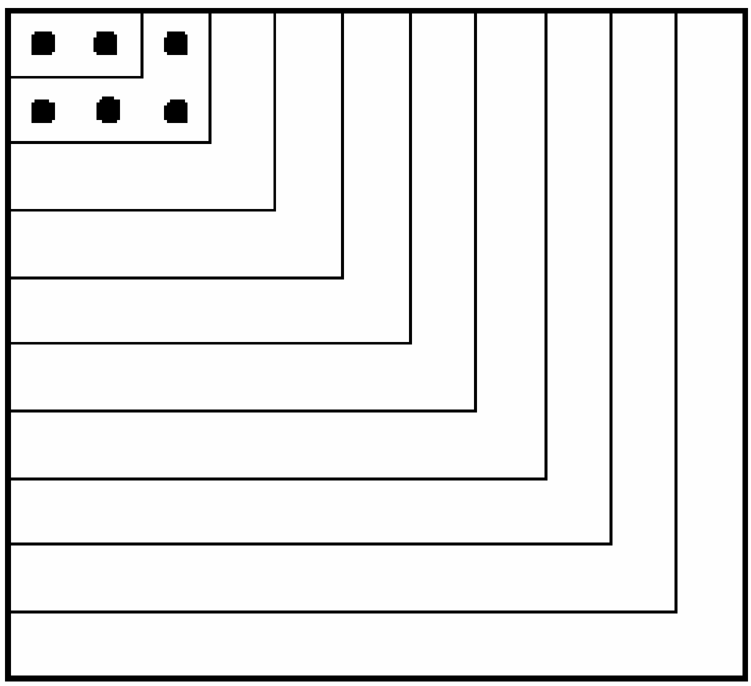
The oblong numbers are found by adding up the terms of the formula, which are the even numbers. Notice they are double the triangular numbers.

Calculate the first 10 oblong numbers by filling in the following table and drawing in the dots to form a series of oblongs from the top left corner.

The even number of new dots around the old oblong is added to the total number of dots in the previous oblong to give the new oblong number for that term.

Term *0 1 2 3 4 5 6 7 8 9 10*

Adding	*Oblong Number*
+ 2	= 2
+ 4	= 6
+ 6	=
+ 8	=
+ 10	=
+ 12	=
+ 14	=
+ 16	=
+ 18	=
+ 20	=



Write here a summary of the first 10 oblong numbers:

Term	1	2	3	4	5	6	7	8	9	10
Oblong Number										

11. Betts' Square Root Approximation – Part 1 ❀ ❀

This method will give an approximation of a square root (√) to at least 1 decimal place.

Calculate the square root of 70 by finding the square numbers above and below, and then write down the square roots of these known perfect squares (the table below indicates perfect squares as underlined values):

Square number just above 70 = **81**, which has the square root **9**.

Square number just below 70 = **64**, which has the square root **8**.

This means that the square root of 70 is above **8** and below **9**.

The square root of 70 is calculated by the whole number of the lowest perfect square, in this case the whole number is **8**, plus a fraction of the remainder.

The remainder is calculated as a fraction of the difference between 70 and the next perfect square below divided by the difference of the square number above and the square number below.
Calculate the top and bottom of this fraction:

Difference of 70 and the square number below = 70 - **64** = **6** (top of fraction)

Difference of the squares above and below = **81** - **64** = **17** (bottom of fraction)

The fraction is (top) ÷ (bottom) = **6** ÷ **17** = **0·3** (to 1 decimal place only)

Thus the square root of 70 = whole number + remainder = **8** + **0·3** = **8·3**.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	**64**	72	80
9	18	27	36	45	54	63	72	**81**	90
10	20	30	40	50	60	70	80	90	100

12. Betts' Square Root Approximation – Part 2 ❀ ❀

The square root of 55 is calculated in two steps. First, by finding the square numbers above and below, then by working out the fraction of the distance between them.

Find and then write down the square roots of these known perfect squares using the table below:

Square number just above 55 = ____, which has the square root ____.

Square number just below 55 = ____, which has the square root ____.

This means that the square root of 55 is above ____ and below ____.

The square root of 55 is calculated by the whole number of the lowest perfect square, in this case the whole number is ____, plus a fraction of the remainder.

The remainder is calculated as a fraction of the difference between 55 and the next perfect square below, divided by the difference of the square number above and the square number below. Calculate the top and bottom of this fraction:

Difference of 55 and the square number below = ___ - ___ = ___ (top of fraction)

Difference of the squares above and below = ___ - ___ = ___ (bottom of fraction)

The fraction is (top) ÷ (bottom) = ___ ÷ ____ = ___ (to 1 decimal place only)

The square root of 55 = whole number + remainder = ___ + ___ = **___.**

<u>1</u>	2	3	4	5	6	7	8	9	10
2	<u>4</u>	6	8	10	12	14	16	18	20
3	6	<u>9</u>	12	15	18	21	24	27	30
4	8	12	<u>16</u>	20	24	28	32	36	40
5	10	15	20	<u>25</u>	30	35	40	45	50
6	12	18	24	30	<u>36</u>	42	48	54	60
7	14	21	28	35	42	<u>49</u>	56	63	70
8	16	24	32	40	48	56	<u>64</u>	72	80
9	18	27	36	45	54	63	72	<u>81</u>	90
10	20	30	40	50	60	70	80	90	<u>100</u>

Chapter 3

Squares and Circles

13. The Square and the Square Root ❁

The square is a 4-sided equi-quadri-lateral with 4 equal sides and 4 right angles.

The diagonal is the exact measurement from one corner to the opposite corner.

Use a ruler to measure the following dimensions:

Diagonal length of the square below = _____________cm

Side (lateral) length of the square = _____________cm

The ratio of the diagonal to the side = diagonal : side

= diagonal ÷ side

= _____________ ÷ ____________

As an approximation, = _______________________ (ratio).

The square root button on a calculator is √.

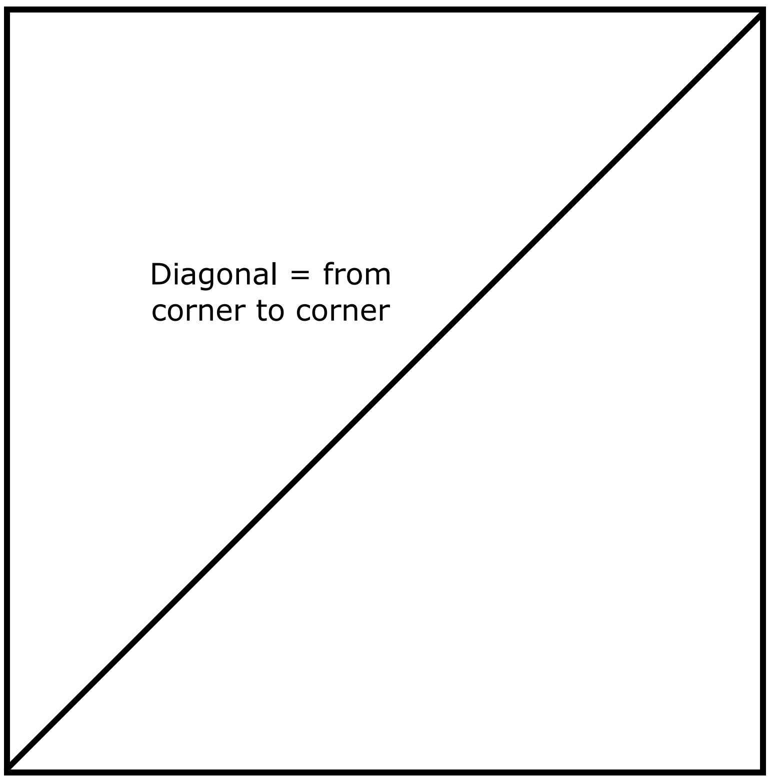
Calculate the square root of 2 by [calculator] 2 √

The square root of 2 = √2 = _____________

The formula for the area of a square = (side length) x (side length)

= __________ cm x _____________ cm

= __________ cm^2.



14. Areas of Squares ❁ ❁

Use a ruler to measure the lengths of the square below. Length = _____ cm.
Calculate the area by length x length = ______ cm x _____ cm = _____ cm^2.

Mark the four mid-points of the side with a dot by using a ruler to find half way. Construct a smaller square inside the first by drawing lines joining the four dots. Use a ruler to measure the length of the second square. Length = cm.
Calculate the area by length x length = _____ cm x _____ cm = _____ cm^2.

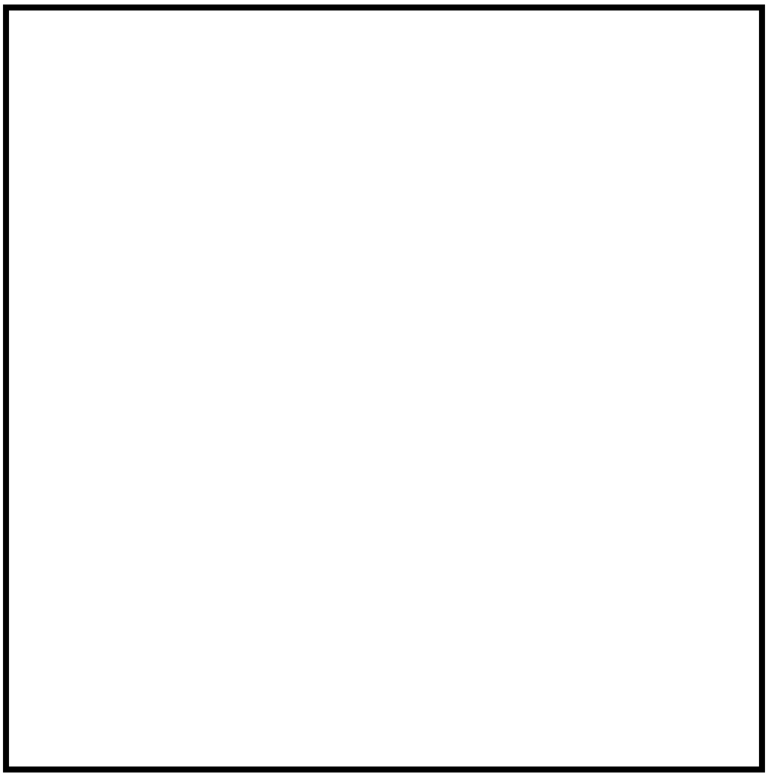
Use a ruler and pencil to connect the corners and the opposite midpoints of the first big square. Draw a third square using the intersection of the diagonals with the second square as the four corners.
Calculate the area by length x length = _____ cm x _____ cm = _____ cm^2.

Mark the four mid-points of the sides of the third square with dots by using a ruler to find half way, drawing lines joining the four dots. Length = __ cm.
Calculate the area by length x length = _____ cm x _____ cm = _____ cm^2.

Draw a fifth square using the intersection of the diagonals with the fourth square as the four corners. Side length of the fifth square = _____cm.
Calculate the area by length x length = _____ cm x _____ cm = _____ cm^2.

The ratio of areas is:	area 1 :	area 2 :	area 3 :	area 4 :	area 5
which is:	______ :	_____ :	_____ :	______ :	_____
Divide all by area 5	______ :	_____ :	_____ :	______ :	1 .

This means that every mid-point square is half as big in area as the square that it is contained in.



15. Pythagoras' Theorem ❀ ❀

The Pythagorean theorem says that for the diagram below, where A, B and C are the squares around a right angle triangle,
the Area of Square C = Area of Square A + Area of Square B.

Use a ruler to take the following measures and prove Pythagoras' theorem.

Side length of Square A = _____ cm

Area of Square A = _____ cm x _____ cm = _____ cm^2 (Area of A)

Side length of Square B = _____ cm

Area of Square B = _____ cm x _____ cm = _____ cm^2 (Area of B)

The sum of square areas = (Area of A) + (Area of B)

= _____ cm^2 + ____ cm^2 = _____ cm^2

Side length of Square C = _____ cm

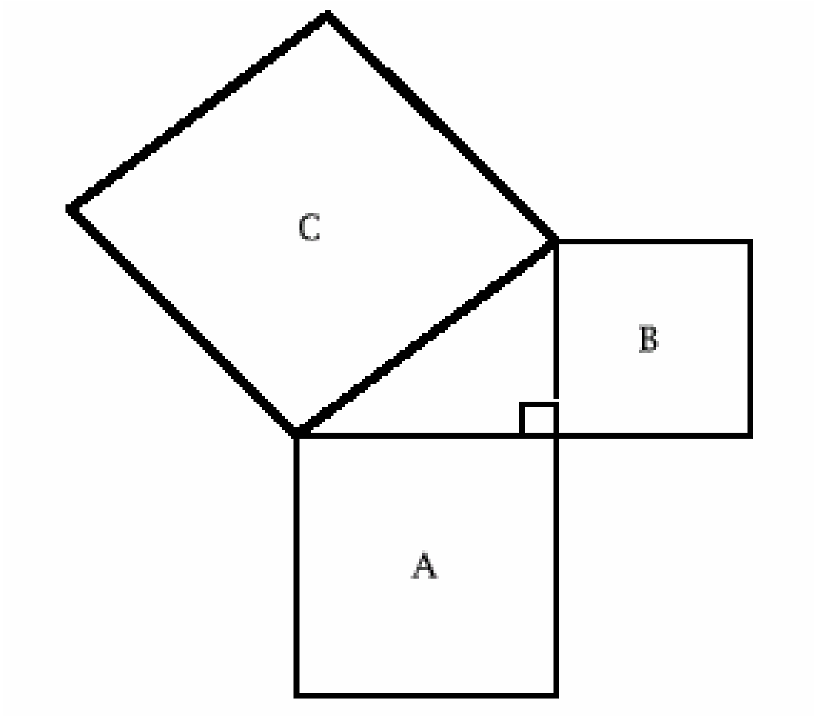
Area of Square C = _____ cm x _____ cm = _____ cm^2 (Area of C)

Is the Area of Square C = the sum of Areas of Square A + Square B? ____

The ratio of side lengths = side A : side B : side C = ____ : ____ : ____

Find the smallest ratio by dividing all 3 side lengths by the smallest of them.
____ : ____ : ____.

Now multiply these by 3: ____ : ____ : ____.



16. The Radius, Diameter and Circumference ❁

Use the diagram below to take your measurements.

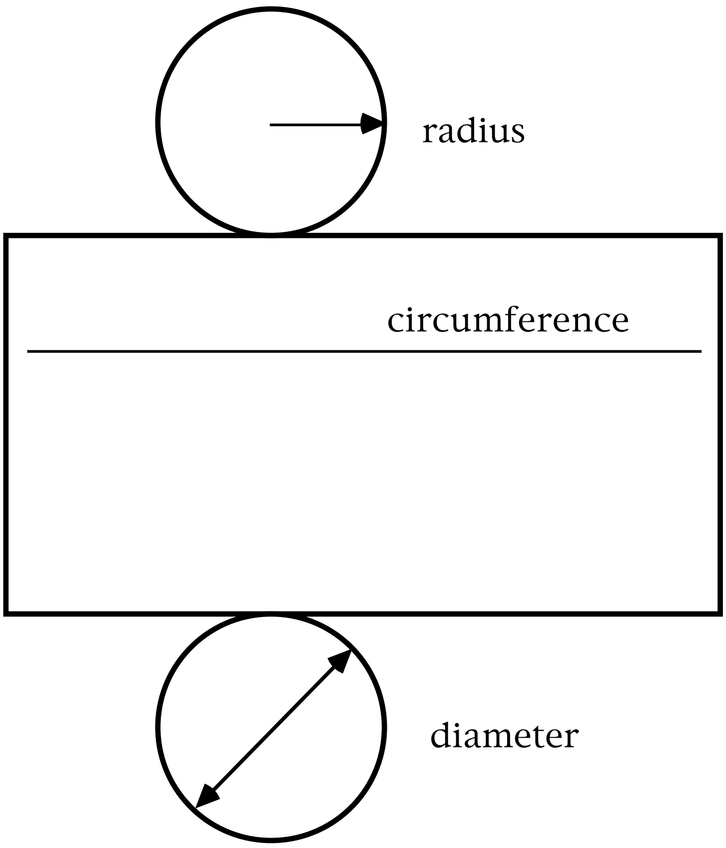
1. The radius is the distance from the centre of the circle to the edge of the circle. The edge is called the circumference of the circle.
2. Use a ruler to measure the radius of the circle, radius = _______cm.
3. The circumference is also known as the distance around the circle.

   Use a ruler to measure the circumference of the cylinder, which in the diagram is the width of the rectangle that the circle is resting on. If you constructed a cylinder with the rectangle, it would wrap exactly around the circle.
4. The diameter of the circle is the measure (meter) across (dia) from the circumference to the circumference, passing through the centre point.

   Use a ruler to measure the diameter of the circle, diameter = _______ cm.

5. Notice that the diameter is exactly twice the length of the radius. The formula for the diameter is: diameter = 2 x radius, or D = 2r.

[MB: MUST MEASURE THESE]



17. The Ratio Pi ❁

The ratio pi (π) is the number of diameters that can fit into the circumference of a circle. Use a ruler and the diagram below to take the following measurements.

It is calculated: π = circumference ÷ diameter

= _______cm ÷ ______ cm

As an approximation, = _________________(ratio).

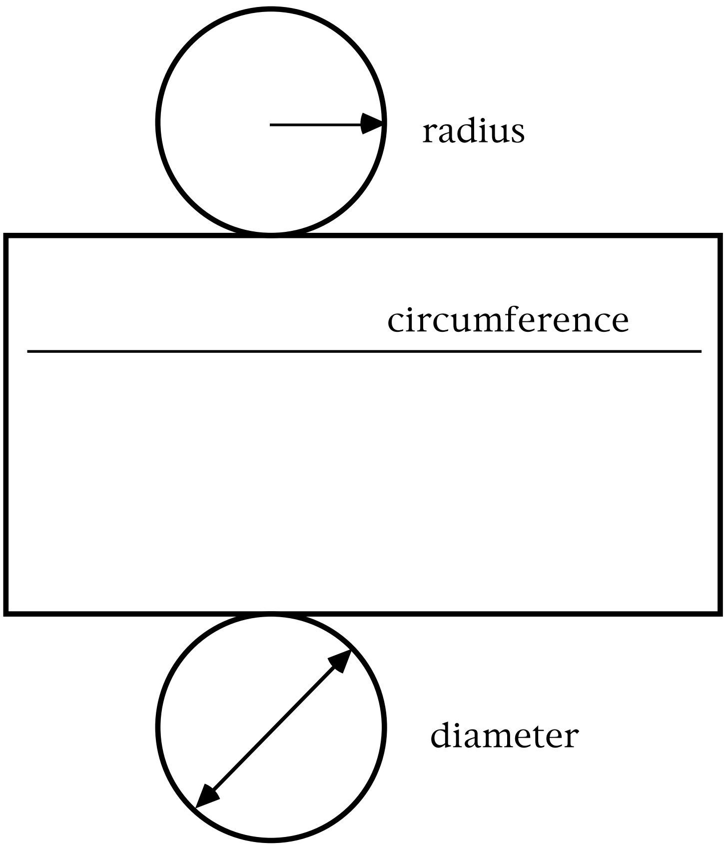
As the measure of the diameter is the same as twice the measure of the radius, calculate the value for pi by using the formula D = 2r.

π = circumference ÷ diameter

= circumference ÷ (2 x radius)

= _______cm ÷ (2 x ______cm)

As an approximation, = _______cm ÷ ______ cm =_______ (ratio).



18. Squaring the Circle ❁

Use a pair compasses to draw a circle so that it fits exactly in the square below, using the centre and circumference dots to guide you.

Use a ruler to measure the radius of the circle, radius = _____cm.

The formula for the area of a circle = πr^2 = π x radius x radius

where π = 3.14159, = 3.14159 x ___cm x ___cm

= ____cm^2.

Calculate the area of a circle whose radius is 1 unit.

The formula for the area of a circle = πr^2 = π x radius x radius

= 3.14159 x ___ x ____ = ____ ($units^2$)

Calculate the area of a square that fits outside a circle with a radius of 1.

If the radius = 1 unit, then the diameter = 2 x radius = 2 x ___ = ___ .

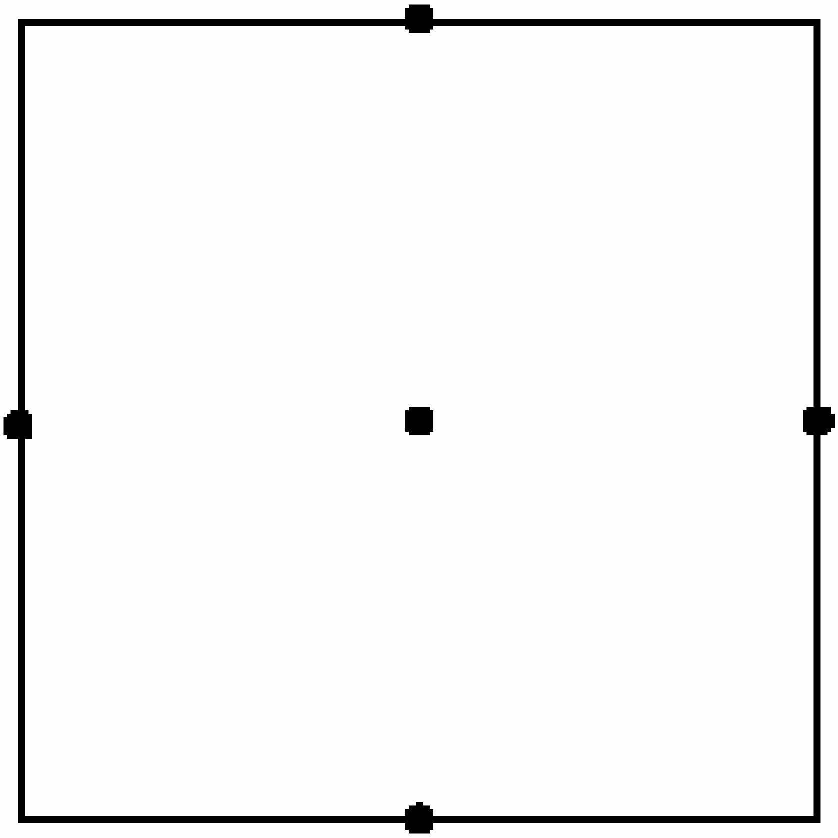
The area of a square is = $(\text{side length})^2$ = (side length) x (side length)

= diameter x diameter = ____ x ____ = _____ $units^2$.

Calculate the percentage that a circle with a radius of 1 occupies in the square of best fit.

Percentage of area = area of circle : area of square = ____ ÷ ____ = ____%.

The area left over from the circle within the square = 100 - ____% = ____%.



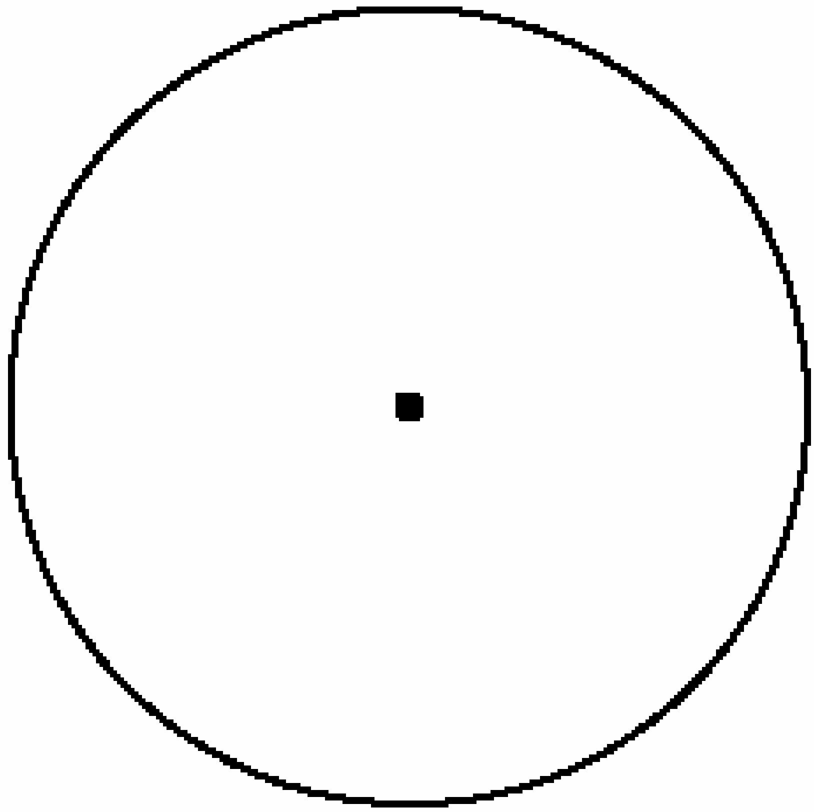
19. Inside the Circle – Part 1 (right angle) ❁

1. Use a ruler to draw a line from circumference to circumference, passing through the centre, labelling the end points A and B.
2. Plot a dot on any part of the circumference and label it point C.
3. Construct the triangle ABC by using a ruler to join the dots A, B, C.
4. Use a protractor to measure the internal angle at C = _______ degrees.
5. Repeat the above exercise for points D, E, F, and record the angles below.

   Angle at point D for triangle ABD = __________ degrees.

   Angle at point E for triangle ABE = __________ degrees.

   Angle at point F for triangle ABF = __________ degrees.
6. A 90 degree angle is called a _____________ angle.



20. Inside the Circle – Part 2 (√3 triangle) ❁

1. Use a ruler to draw a line from circumference to circumference, passing through the centre, labelling the end points A and B.
2. Set the compass point on point B and draw an arc through the centre to the circumference and label where it intersects the circumference point C.
3. Use a ruler to construct the triangle ABC and measure the side lengths.

   Length of chord-line BC = __________ cm

   Length if diameter line AB = __________ cm

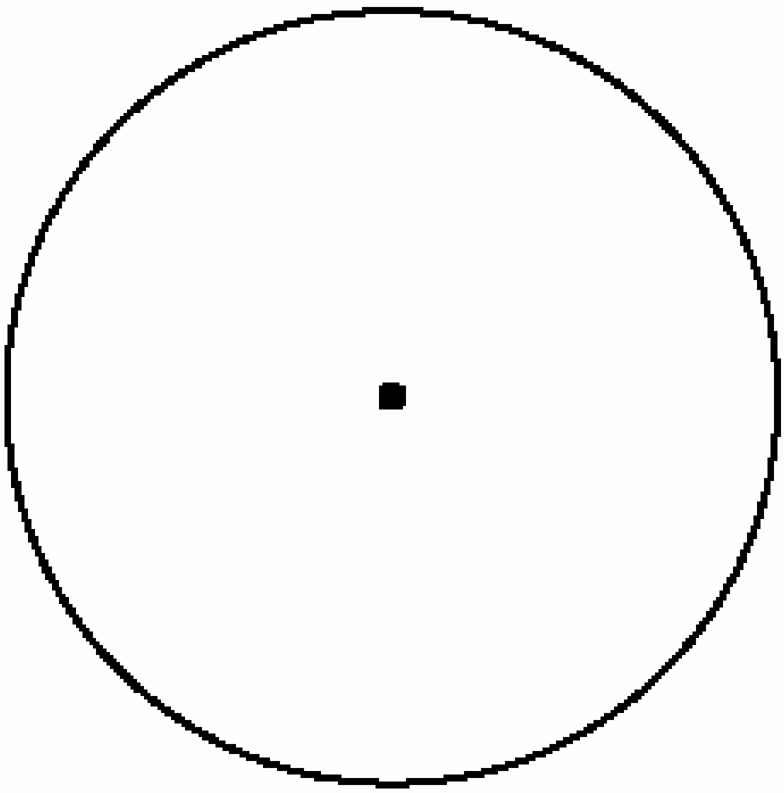
   Length of chord-line AC = __________ cm
4. The ratio of lengths of BC : AB : AC = ___ : ___ : ___.

   Divide all of them by the smallest number = 1 : ___ : ___.
5. Pythagoras' theorem in this case would be: $(AB)^2 = (AC)^2 + (BC)^2$

   Rearranging this, for the line segment AC: $AC = \sqrt{((AB)^2 - (BC)^2)}$

   $= \sqrt{(4 - 1)} = \sqrt{3}$.

   Use a calculator to find √3 by [calculator] 3 √ = __________ .

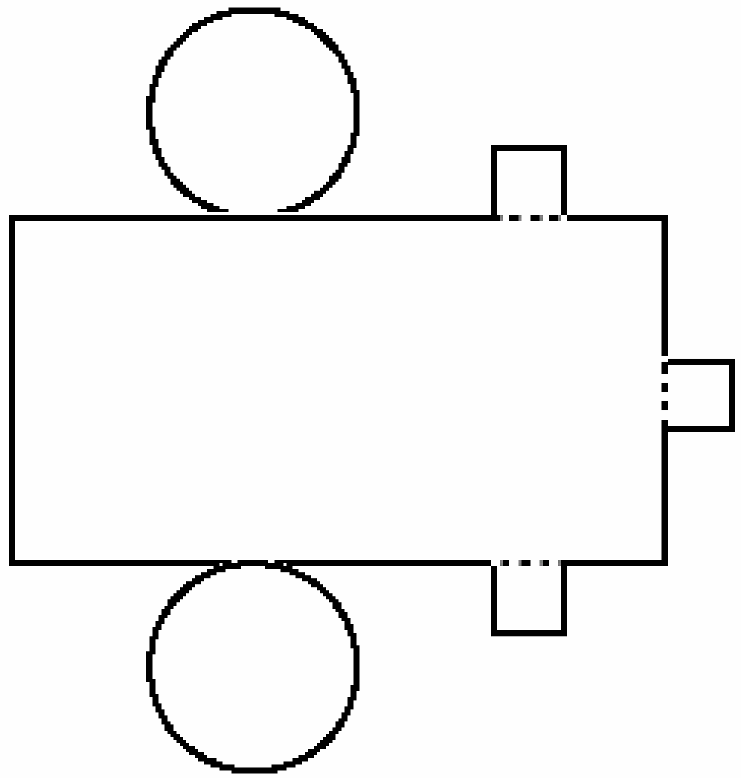


Chapter 4

Solid and Plane Geometry

21. Build a Circular Cylinder ❀

1. Photocopy this page or use another sheet of paper to make a tracing.
2. Use scissors to cut out the whole outside shape from the copy.
3. Fold along the dotted lines to create the flaps and edges.
4. Build the circular cylinder by wrapping the rectangle around the circles.
5. Use sticky tape to tape down the cylinder flaps over the circular ends.

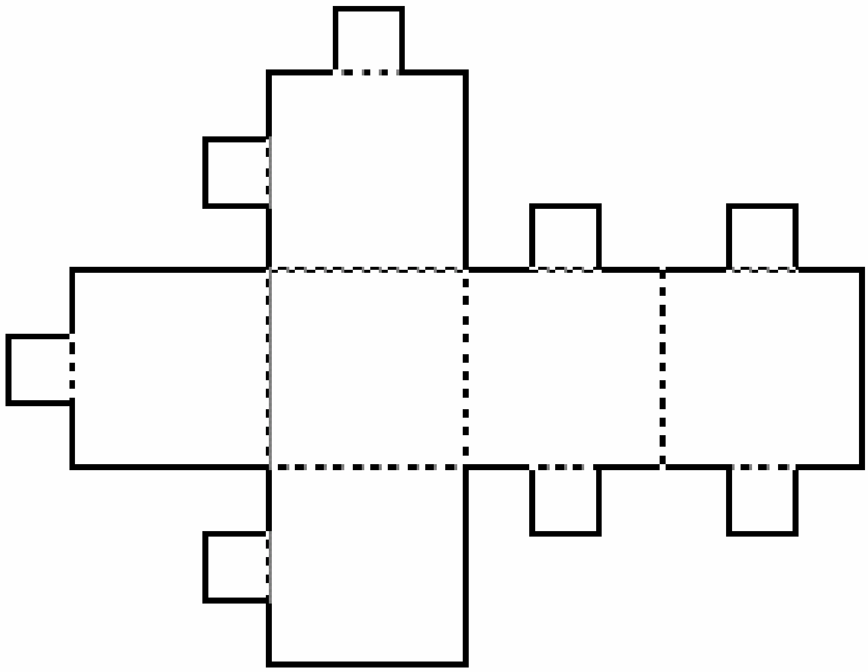


22. Build a Hexahedron (die cube) ❀

1. Photocopy this page or use another sheet of paper to make a tracing.
2. Use scissors to cut out the whole outside shape from the copy.
3. Fold along the dotted lines to create the flaps and edges.
4. Build the cube by wrapping the side panels around the base square.
5. Use sticky tape to tape down the panel flaps over the other panels.
6. Make a die cube (dice is plural) by drawing big dots on the panels.

   Draw the 1, 2, 3 dot panels first and make them share the same corner.

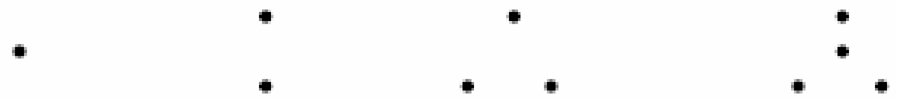
   Put 6 opposite 1, 5 opposite 2, and 4 opposite 3, as opposite panels add to 7.

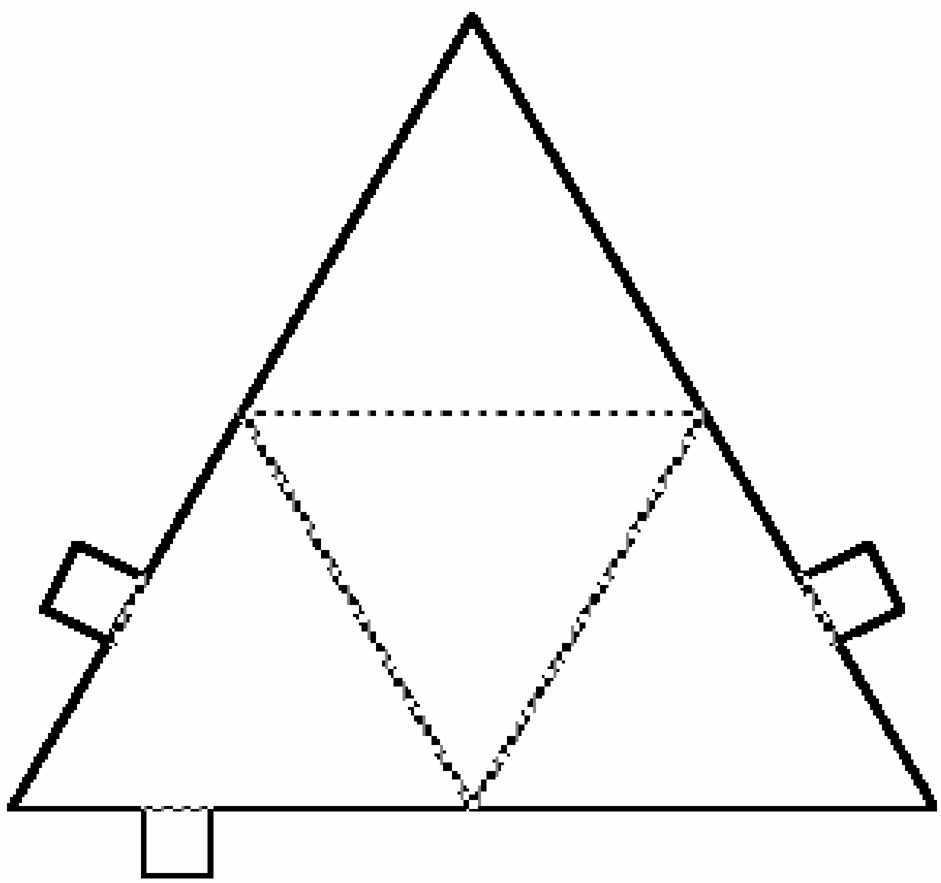


23. Build a Tetrahedron (triangular pyramid)

1. Photocopy this page or use another sheet of paper to make a tracing.
2. Use scissors to cut out the whole outside shape from the copy.
3. Fold along the dotted lines to create the flaps and edges.
4. Build the tetrahedral by wrapping the triangles around the base triangle.
5. Use sticky tape to tape down the triangles over the other triangles.
6. Make a die pyramid by drawing big dots on the triangular panels.

   Draw the 1, 2, 3, 4 dot patterns in any order, with a design like this:

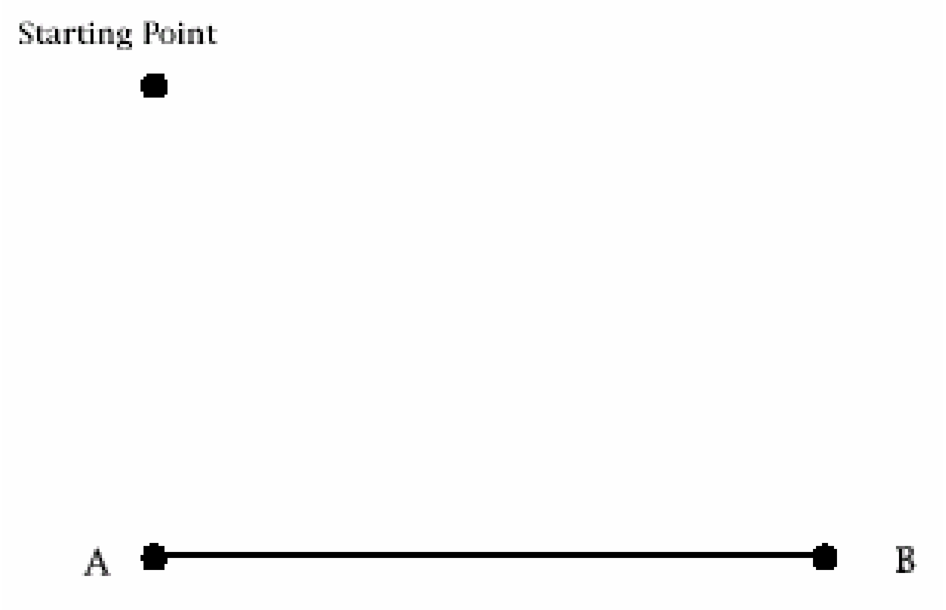




24. Bisect a Straight Line

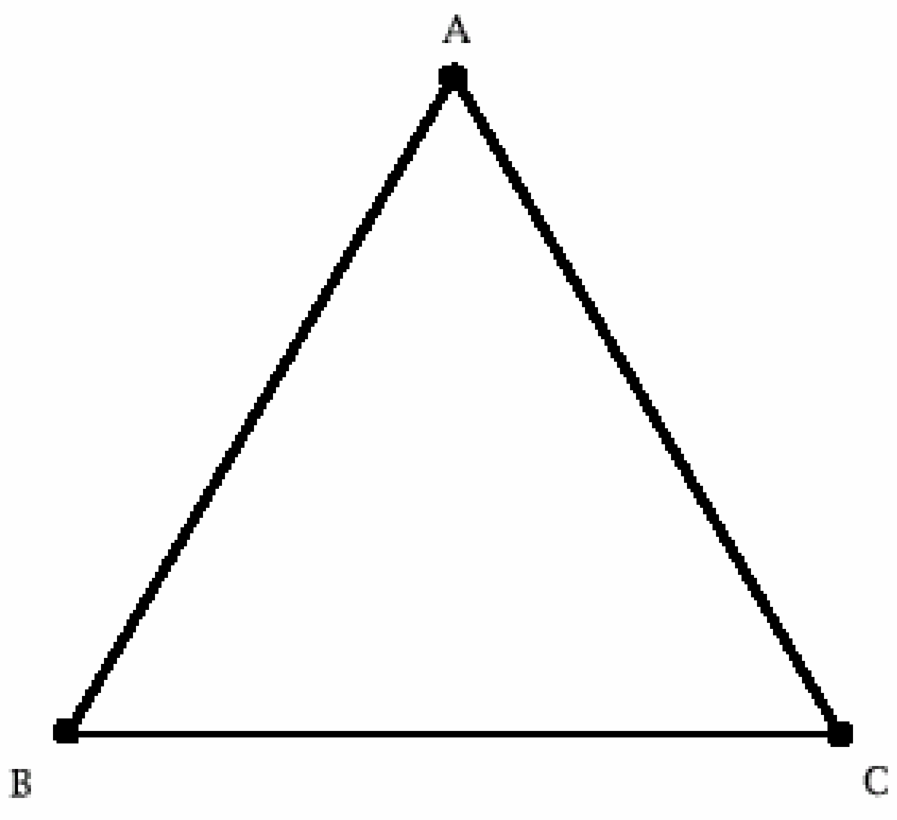
Find the half-way (mid) point of the line below by using a compass to draw a semicircle (a half circle) around end points A and B.

1. Put your compass point on A and with the pencil at the starting point draw downwards until a semicircle is drawn. Repeat this for point B.
2. The semicircles will overlap twice, once above and once below the line AB. Use a ruler to draw a line through these two points of overlap.
3. The point where this line intersects line AB is the mid-point of AB. Label the mid-point of AB as point C by drawing a small dot and a letter C.



25. Bisect a Triangle ❀

1. Use a compass to draw a semicircle around A by putting the compass point on A and the pencil starting two thirds of the way towards B.
2. Draw a semicircle downwards crossing the lines towards B and C. Use the same compass width to draw overlapping semicircles around B and C.
3. Use a ruler to mark the bisection mid-points and draw lines from them to the opposite vertex, ie from mid-point of BC to point A, mid-point AB to C and midpoint AC tp B.
4. Notice that each bisecting line connects with the other vertex of the triangle when all the sides are the same length. This only happens in an equilateral triangle.
5. Notice that the centre of the triangle is found by the intersection of the three bisecting lines from each corner to the mid-point of the opposite sides.

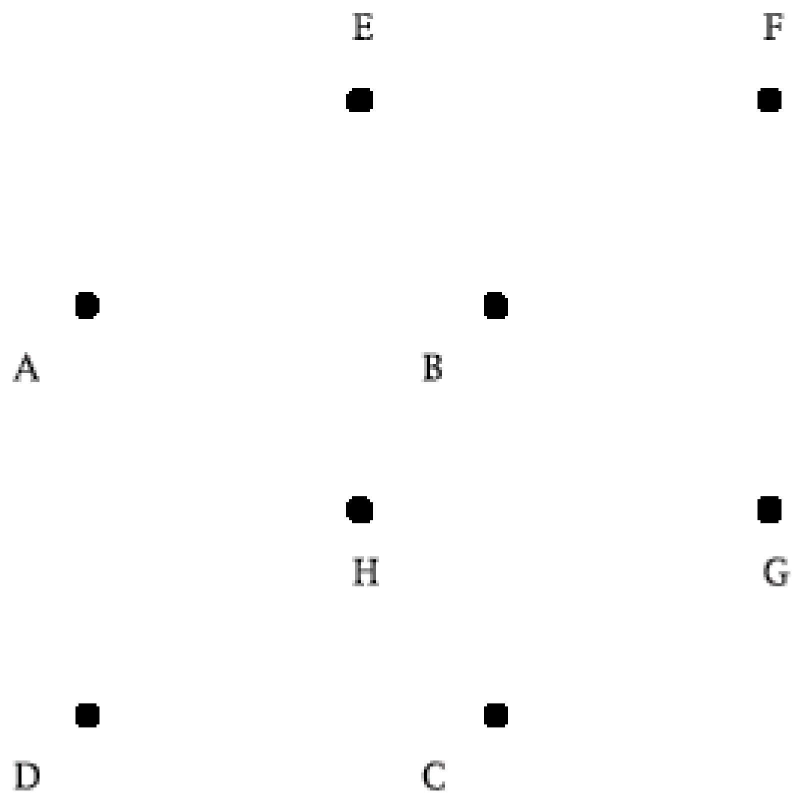


26. Construct a 3D Cuboid ❁

1. Use a ruler to join the points by the following constructions:

connect the rectangle ABCD	(4 edges)
connect the rectangle EFGH	(4 edges)
connect the rectangle AEHD	(2 edges)
connect the rectangle BFGC	(2 edges).

2. A cube is made of 6 panels (sides), 8 corners (points) and 12 edges (lines).
3. A regular cube or equilateral (equalsided) cube has all sides the same length.



27. Internal Polygon Angles ❀ ❀

Use a protractor to measure the internal (inside) angles of the following regular polygons, and then multiply that number by the number of corners of that polygon to calculate the total internal angle for that polygon.

Internal Angle of a Straight Line = __ degrees x 2 sides = _____ degrees

Internal Angle of a Triangle = __ degrees x 3 sides = _____ degrees

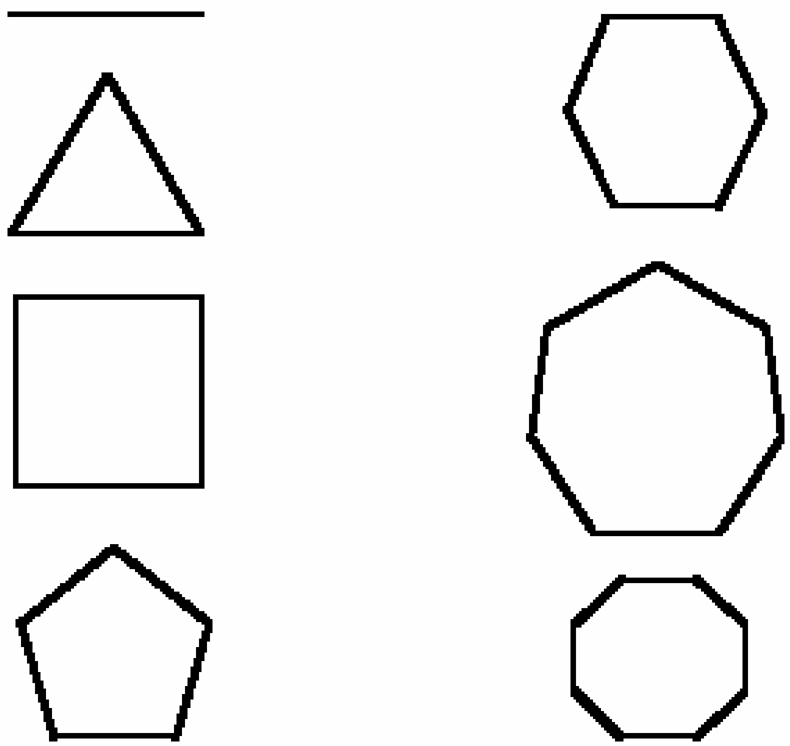
Internal Angle of a Square = __ degrees x 4 sides = _____ degrees

Internal Angle of a Pentagon = __ degrees x 5 sides = _____ degrees

Internal Angle of a Hexagon = __ degrees x 6 sides = _____ degrees

Internal Angle of a Heptagon = __ degrees x 7 sides = _____ degrees

Internal Angle of a Octagon = __ degrees x 8 sides = _____ degrees



28. Internal Angle of a Circle

1. Draw 3 dots anywhere on the circumference of the circle below, and label them points A, B, and C.
2. Use a ruler to make a fourth dot exactly opposite one of the 3 dots by ruling through the centre point, and label this point D.
3. Draw the quadrilateral of the 4 dots by joining the points A, B, C, and D.
4. Use a protractor to measure the internal angles at each point:

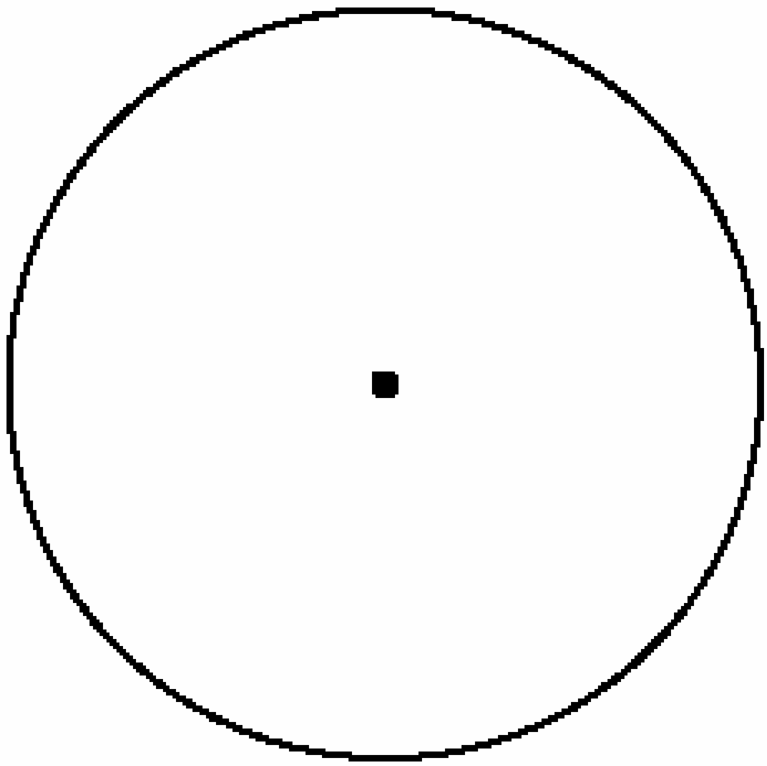
   Internal angle at point A = ____________ degrees

   Internal angle at point B = ____________ degrees

   Internal angle at point C = ____________ degrees

   Internal angle at point D = ____________ degrees

5. Use a calculator to work out the sum of the internal angles of quadrangle ABCD = ______________ degrees.
6. Connect the points A, B, C to make a triangle. Use a protractor to add up these 3 internal angles for triangle ABC = __ + __ + __ = ________ degrees.



29. Construct a Pentagon ❀ ❀ ❀

1. Use a protractor to measure 72 degrees above the line AB to the left of A and make a small dot against the protractor.
2. Use a ruler to draw a line from A through this point.
3. Use a compass to make a circle around A with the circumference at B.
4. Label the intersection of this circle with the line away from A point C.
5. Place the protractor at point C along time AC and repeat the process, making a dot at 72 degrees to the left of C.
6. Construct a pentagon by repeating the process clockwise until reaching point B.
7. Pentagon means ___ corners. A regular pentagon has 5 equal sides of: [calculator] 360 ÷ 5 =________degrees.

Chapter 5

Sacred Geometry

30. Circles of the Heart in 2D ❁

1. Use a pencil to draw a circle around the circumference of a 20 cent coin in the middle of the blank part of the page below.
2. Guess right now how many circles of the same size you think would fit around the edge of the central circle so that the circles are just touching.

   I guess that I could draw ____ circles around the original circle.
3. Using the same coin and pencil, draw a circle just touching the original circle.
4. Keep drawing circles around the circumference of the original circle, each circle just touching the circles next to it.
5. Put a dot in the centre of all of the outer circles as exactly as you can.
6. Use a ruler to join the dots of the outer circles.
   This shape is called a hexagon, which means _______ corners.
7. Use a ruler to make two triangles by joining every second dot (skip a dot).

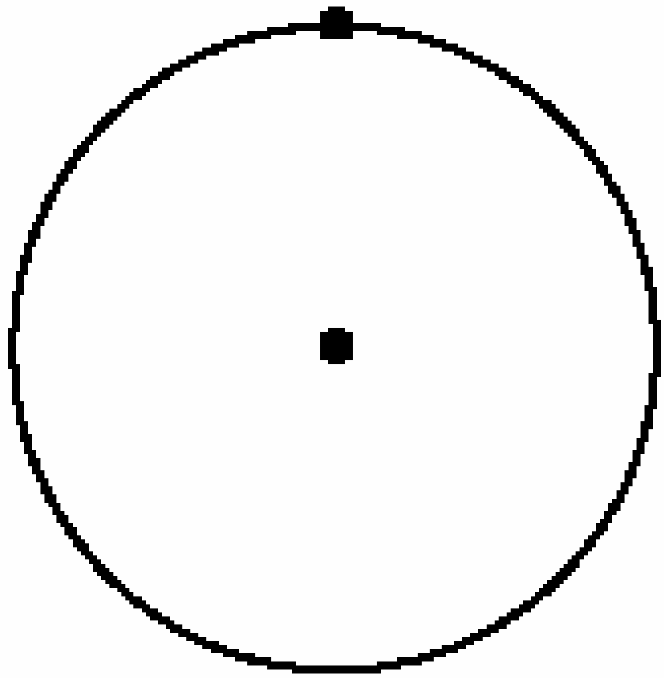
31. Spheres of the Heart in 3D ❀ ❀

1. Consider the previous exercise but think about using ping pong balls (spheres) instead of a coin. Remember that 6 fit around the 'equator'.
2. Guess right now how many spheres of the same size you think would fit around the surface of the central sphere so that the balls are just touching.

   I guess that _____ spheres could be placed around the original sphere.
3. Use sticky tape to stick 6 spheres (balls) around 1 central sphere (ball) around the 'equator' of the 'Earth'.
4. Look at the model and consider how many more spheres will fill each of the 'poles' of the 'Earth'.
5. Use sticky tape to stick ping pong balls to fill the two 'pole' positions.
6. In the space below draw what you think the spheres would look like.
7. There are _____ spheres around the equator and ____ at each pole.
8. The total number of 3D spheres around a central one sphere is _____.

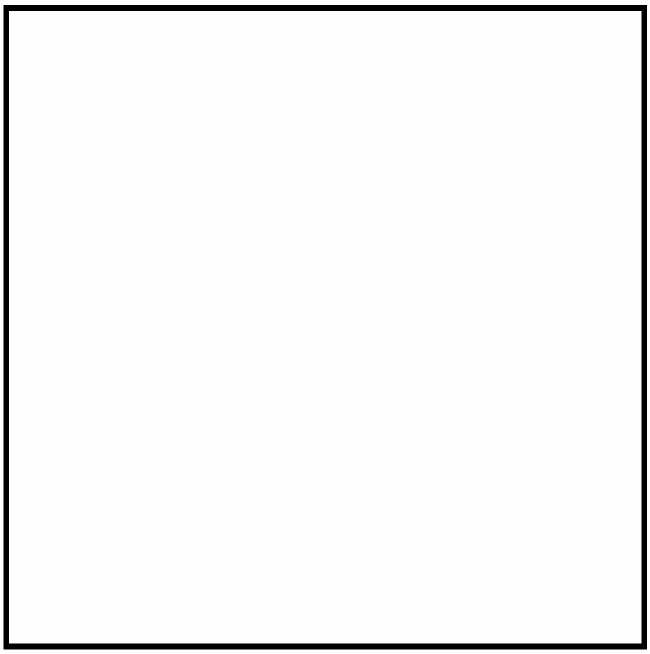
32. The Flower of Life ❀ ❀

1. Use a compass to draw a circle around the top starting point, so that it passes through the centre. Start with the point on the top and the pencil in the centre.
2. Draw another circle with its centre on the point where the new circle and the circumference of the original circle intersect. Be sure to keep the width of the compass the same as the width of the first circle you drew.
3. Use the point where this circle intersects with the previous circle that you drew as the centre for the next circle.
4. Keep drawing circles around the original one until you have six new circles.
5. This pattern is many thousands of years old and was known to the ancient Egyptians as the Flower of Life.



33. Perfect Twelfths of a Circle

1. Use a ruler and pencil to find the centre of the square by drawing diagonal lines.
2. Set your compass point in the centre and draw a circle that best fits within the square.
3. Use the same radius on the compass to draw quarter circles from each corner by putting the point in the corner and arcing.
4. Use the same radius on the compass to draw a semicircle along each side of the square by putting the pencil in a corner and the point on the edge of the square and inscribing half a circle to the other corner along that side.
5. Mark with a small dot the twelve intersections of the arcs on the big circle, starting with the four midpoints of the square.
6. Notice that these twelve marks are exactly evenly spaced like the hours on a clock.



34. Complex Geometric Construction

1. Use a ruler and pencil to find the centre of the square by drawing diagonal lines.
2. Set your compass point in the centre of the square and draw a circle that best fits within the square so that it just touches it.
3. With the compass at the same radius inscribe semicircles along each side by placing the pencil in a corner and the point on the line.
4. Set the compass radius to the side length of the square and place the point in the middle of the top side and inscribe an arc from the left side of the square to the right side.
5. Construct an equilateral triangle by using a ruler to draw a straight line between the ends of this arc, and then draw two lines from each end to the midpoint of the top side.
6. Use a ruler to draw a hexagon from the intersections of the arcs and the circle starting at the midpoint of top side. Construct a hexagram with two triangles from the corners of the hexagon.
7. Set the compass radius to the length of the inside center edge of one of the small triangles of the hexagram and inscribe a circle in the centre of the hexagram.
8. Set the compass radius to the length of one of the side edges of one of the small triangles of the hexagram and inscribe a circle around the four corners of the square.
9. Set the compass radius to equal the length of the square and then inscribe a circle with the point in the centre of the square and the pencil around the whole thing.

35. Perfect Packing (rhombus) ❀

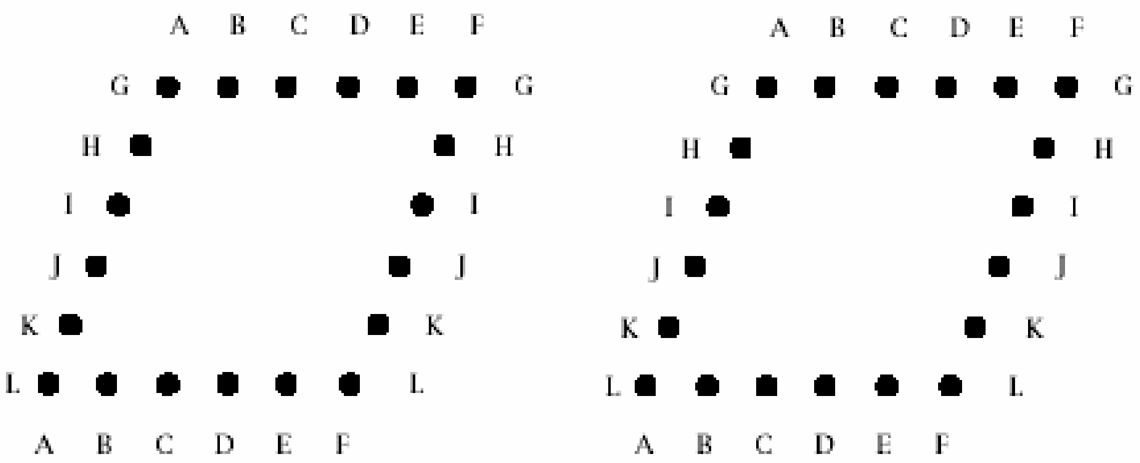
A 'perfectly packed' shape has no spaces between its identical interior shapes.

By using a ruler and pencil to draw lines parallel to the edges of the rhombus. See how the rhombus creates many smaller and perfectly-packed versions of itself.

1. Connect the letters AA, BB, CC, DD, EE, and so on in both the rhombii.
2. Connect the following in the second rhombus only:

from left side upwards to top: HB, IC, JD, KE, LF, BH, CI, DJ,EK.

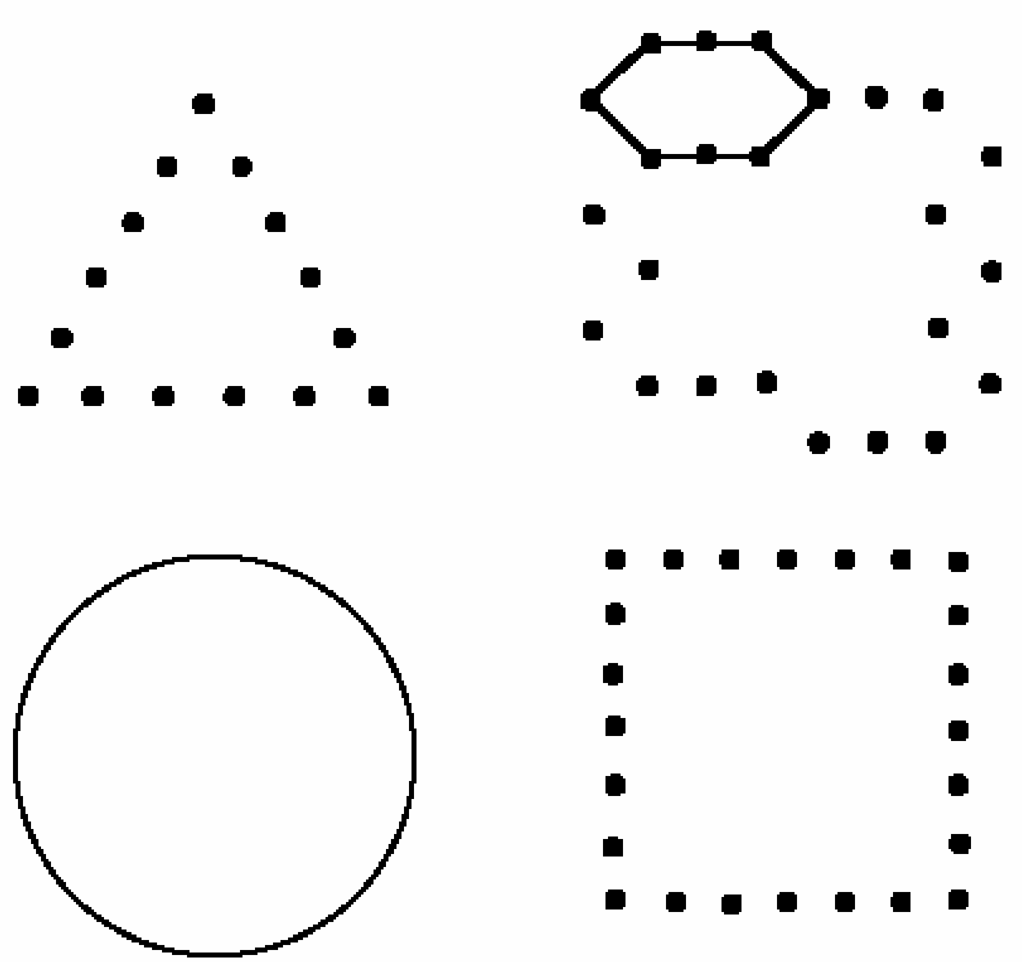
from left side downwards to bottom: KB, JC, ID, HE, GF, BK, CJ, DI, EH.



36. Perfect Packing (polyhedra) ❁

Use a ruler and pencil to create perfect packing inside the following polyhedra.

1. Draw parallel lines inside the outlines of the square and triangle in the same fashion as in the rhombus in the previous exercise.
2. Use hexagon units as shown in the example for the honeycomb polygons.
3. Use a pencil and a five cent coin to see how many circles you can fit in the big circle.



37. Construct a Golden Rectangle ❁ ❁

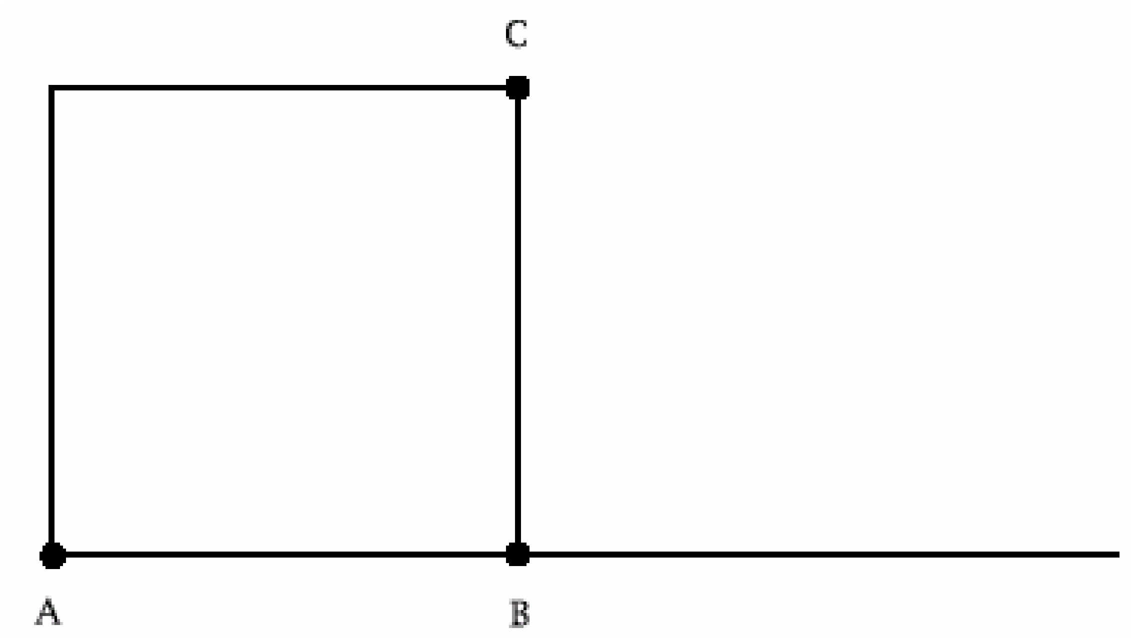
1. Use a compass to draw circles around A and B so that they overlap by making the radius about three-quarters of the distance between AB.
2. Use a ruler to draw a line through the intersections of the circles to bisect the top and bottom of the square. Label the midpoint of AB the point D.
3. Use a compass and put the point on point D and the pencil at point C.
4. Swivel the compass clockwise as you draw an arc from C through the line AB.
5. Label the intersection of this arc and the line AB point E.
6. Use pencil and ruler to extend the top of the square past the point E.
7. Set the compass to the distance of AE. Put the compass point on CD and draw an arc crossing the line to the right of C. Connect this intersection with a line to point E.
8. Use a ruler to measure the following distances:

   Measure of A to E = _____ cm

   Measure of A to B = _____ cm

   Ratio of AE : AB = AE ÷ AB = _____cm ÷ _____cm = _____ (ratio).

   This approximation ratio is called the Golden Ratio = ___________ .



38. Golden Rectangle Reduction ❁

1. Use a ruler to measure the lengths and widths of rectangles A, B, C, and then calculate the ratios of length : width of each one to 1 decimal place.

   Ratio (A) = length of A ÷ width of A = __ cm ÷ ___ cm = ____ (ratio).

   Ratio (B) = length of B ÷ width of B = __ cm ÷ ___ cm = ____ (ratio).

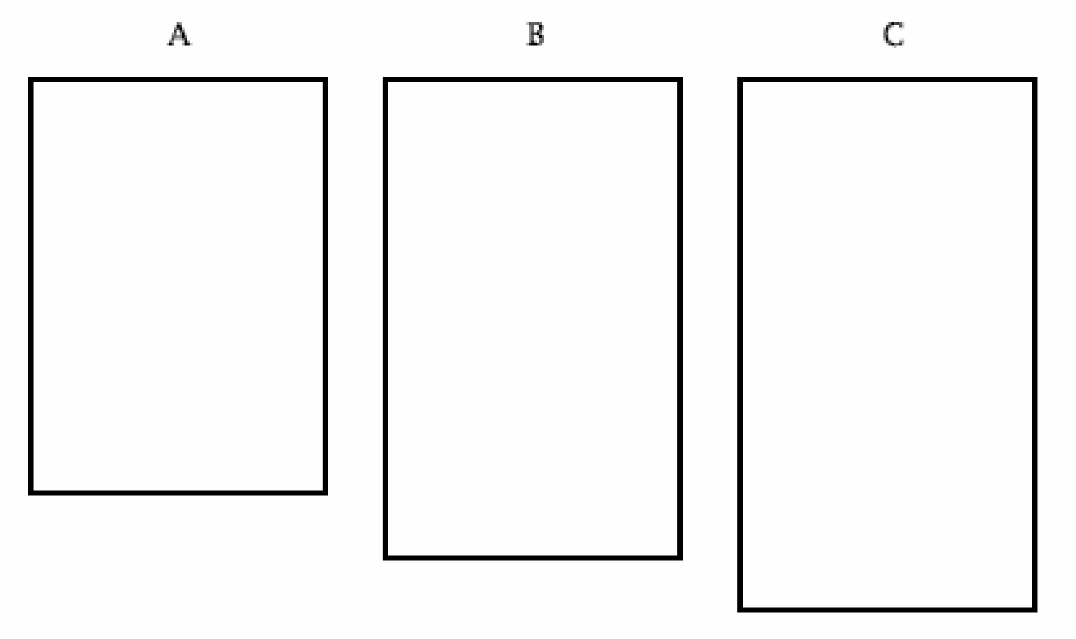
   Ratio (C) = length of C ÷ width of C = __ cm ÷ ___ cm = ____ (ratio).

2. Use a ruler to draw in a square in the top part of each rectangle by making the length of the square down the same as the width of the rectangle across so that it fills up the top part.

3. Draw in squares in the end of each of the parts of the rectangle which is left after the square.

4. Repeat this process of drawing in squares filling up the ends until there is no space left.

5. Notice that rectangle B is a Golden Rectangle, (ratio = 1.6) and keeps on reducing with perfect copies of the same ratio.



39. The Fibonacci Sequence ❀ ❀

1. Starting with 1, 1, keep adding the last 2 numbers of the previous equation to get the next number in the sequence.

1	+	1	=	_______	for example,	1 + 1 = 2
1	+	_______	=	_______	and	1 + 2 = 3
_______	+	_______	=	_______	and	2 + 3 = 5, and so on.
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		
_______	+	_______	=	_______		

2. A summary of the first 14 terms of the Fibonacci sequence is:

1, 1, ____,____,____,____,____,____,____,____,____,____,____,____.

Term: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.

3. Use a calculator to calculate the following ratios of terms in the sequence:

term 2 ÷ term 1	=	__________	÷	__________	=	__________
term 3 ÷ term 2	=	__________	÷	__________	=	__________
term 6 ÷ term 5	=	__________	÷	__________	=	__________
term 10 ÷ term 9	=	__________	÷	__________	=	__________
term 14 ÷ term 13	=	__________	÷	__________	=	__________

4. Use a calculator to calculate the Golden Ratio phi (Φ) = ($\sqrt{5}$ + 1) ÷ 2

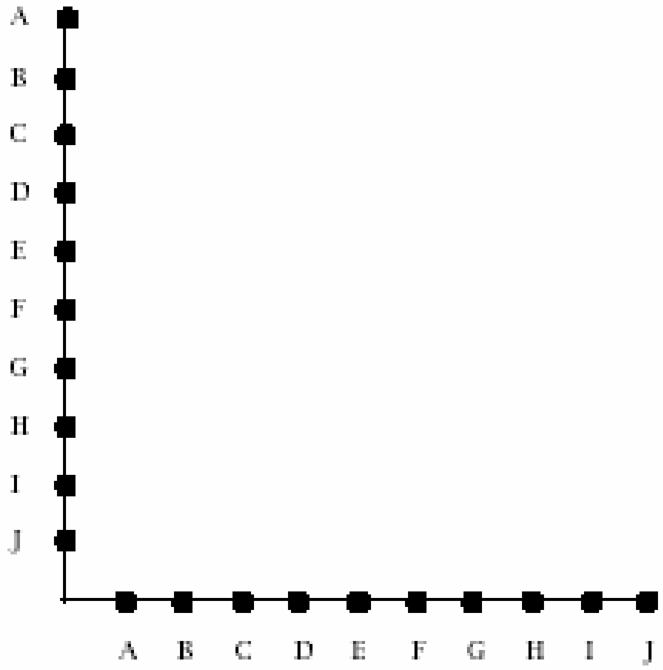
[calculator] 5 √ + 1 = _________ ÷ 2 = _________ = Φ

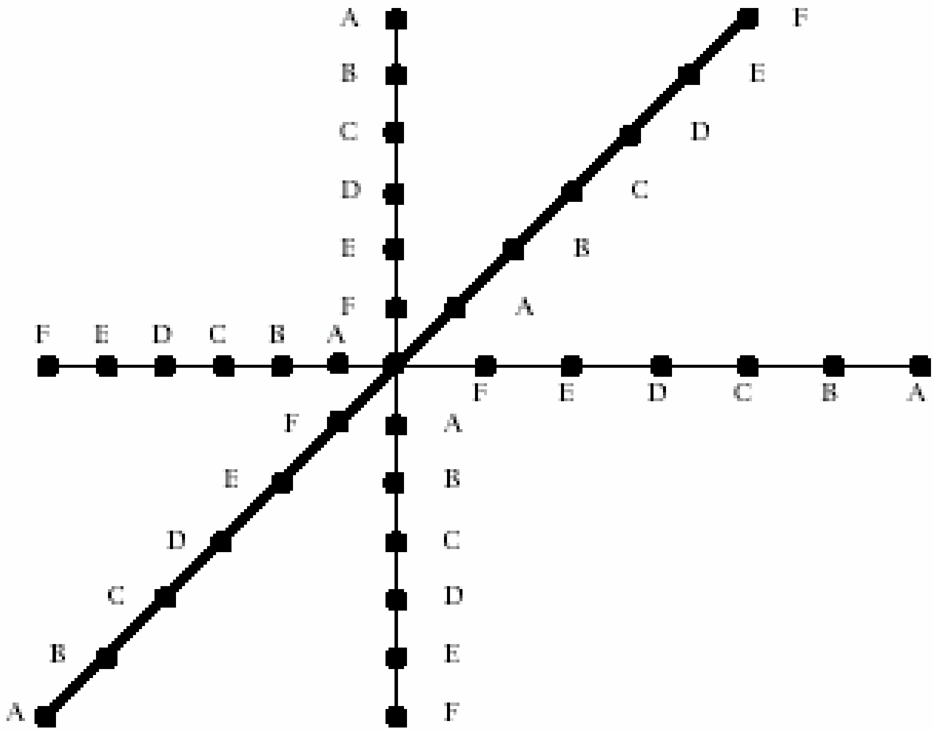
Chapter 6

Geometric Patterns

40. Straight Line Parabola and Hyperbola

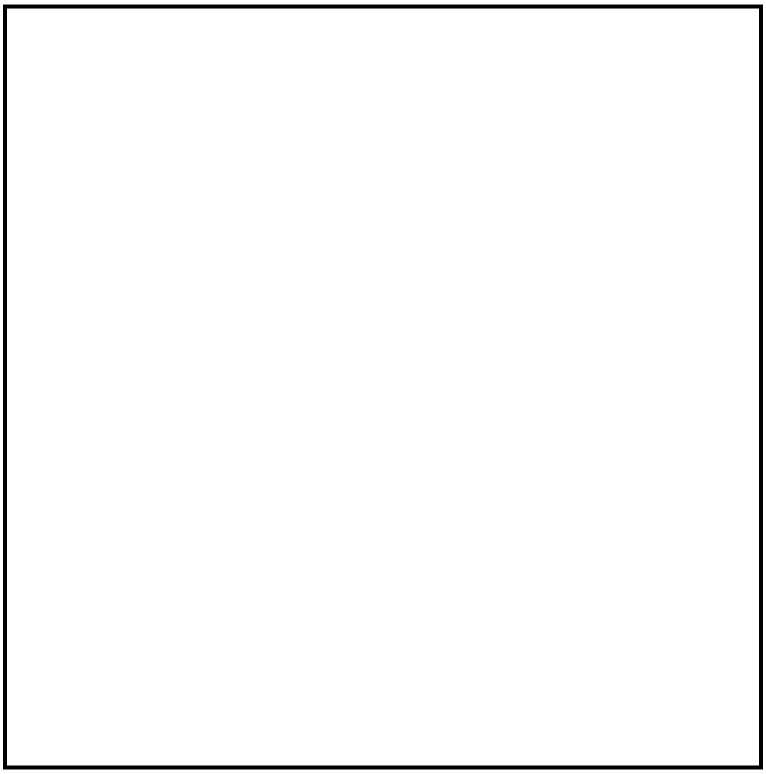
Use a ruler to connect the letters AA, BB, CC, DD, EE, FF and so on.
Connect the letters in the six sections of the second graphs in the same way.





41. Geometric Reduction

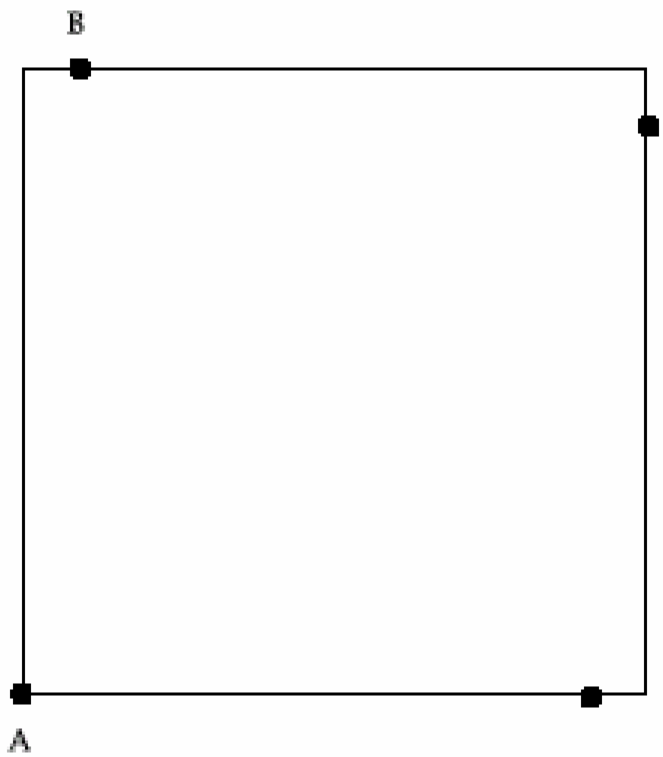
1. Use a ruler to measure the lengths of each side of the square below.
2. Use a pencil to make a dot half-way along each side of the square and draw two lines connecting the opposite midpoints.
3. Use a ruler and pencil to join these four dots together to form another square.
4. Find the mid-points in the sides of the new smaller square by drawing diagonal lines from the corners of the original square.
5. Mark the mid-points with a dot, and join the dots to create a new square. Repeat the process.
6. You will see that the space between each layer of squares forms a set of four triangles in the corners around the inside square. Use coloured pencils four to colour in each set of triangles with the same colour.



42. Linear Rotation and Translation ❁

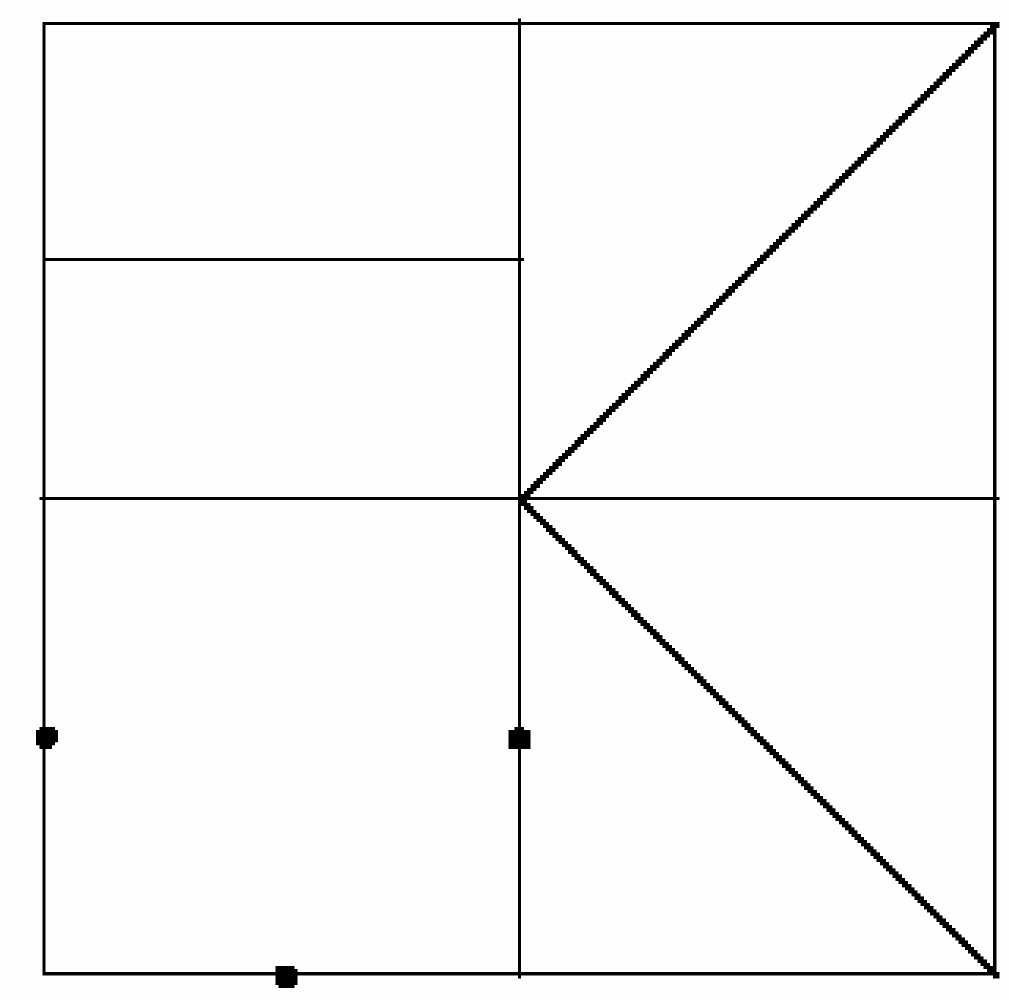
1. Use a ruler to draw a straight line starting at A and finishing at B. Point B is about 1 cm to the right of vertical, or a clockwise rotation of about 5 degrees from vertical.
2. Keeping your pencil on the paper, draw the next line to the next dot.
3. The next line starts where the last line ends, and follows the border clockwise, always 5 degrees to the right of the line already there (starting with the border).
4. Spiral clockwise inwards 5 degrees each time and keep it as neat and accurate as possible.

   There are ________ spirals inwards because there are ________ borders.



43. Contracting Mid-point Spirals ❀ ❀

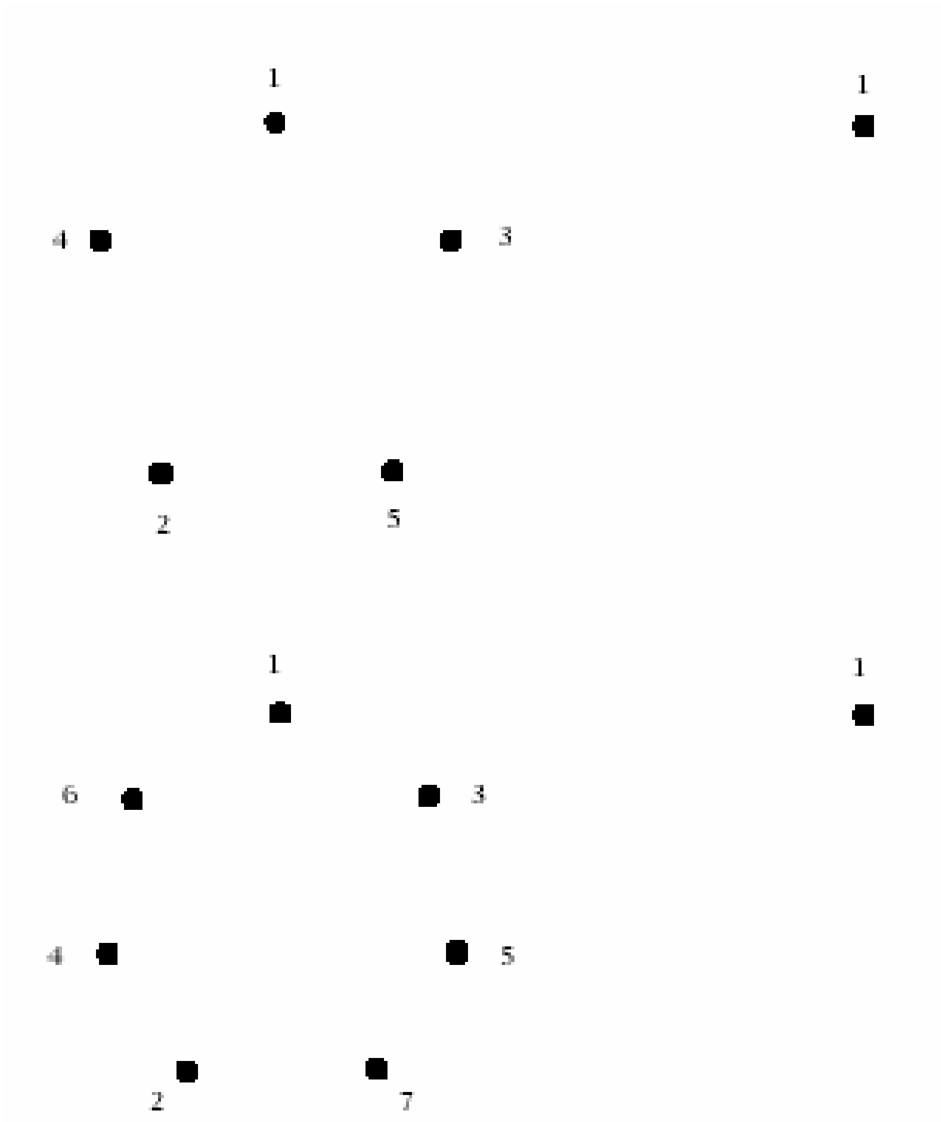
1. Top left square: Use a ruler and pencil to keep dividing the top left section into halves, always taking the top box or the left box, halving the space every time.
2. Bottom left square: Use a ruler and pencil to draw a line from the centre of all four squares to the mid-point of the left side, then the mid-point of the bottom, then the mid-point of the right side, and then the mid-point of the line you drew first. Keep drawing a line from the last one to the mid-point of the next left side.
3. Top right square: Draw a line from the bottom right corner of the square to the middle of the diagonal line, then draw to the right hand side. Then draw to the middle of the remaining diagonal top left, then to the right side again. Repeat into the top right corner.
4. Bottom right square: Draw a line from the middle of the diagonal to the bottom left corner, then use the left half. Draw a line dividing it into halves again, take the left half again or the one closest the centre (which is the top). Spiral each line clockwise and inward.



44. Construct a Pentagram and Heptagram

1. Use a finger to trace the pattern from points 1-2, 2-3, 3-4, 4-5, 5-1.
2. Now use a pencil to connect the points 1-2, 2-3, 3-4, 4-5, 5-1.
3. Repeat the drawing in the space to the right by copying it freehand with a pencil.
4. Repeat the above exercise for the heptagram below, from points 1 through 7 and back to 1.

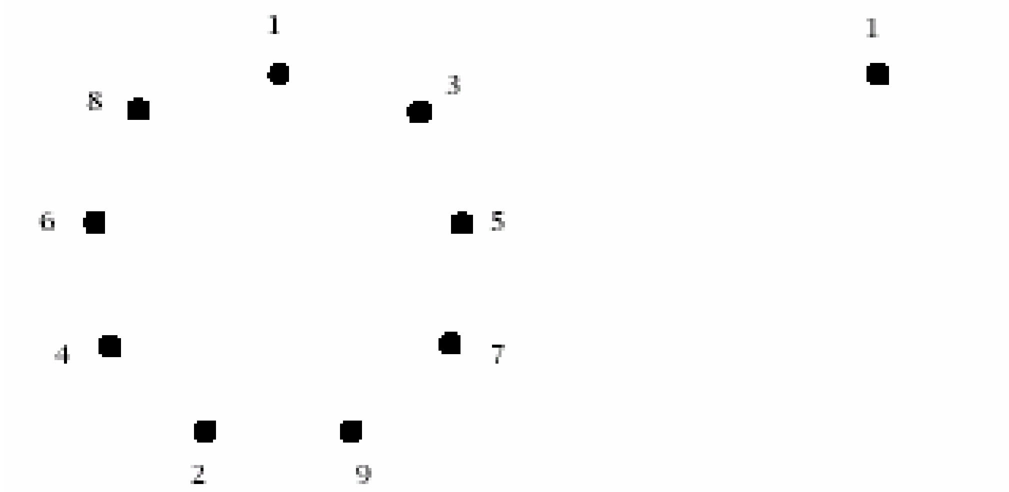
   Notice that the odd numbers start at the top and move clockwise down to the right, and the even numbers start at the bottom and move clockwise up to the left.

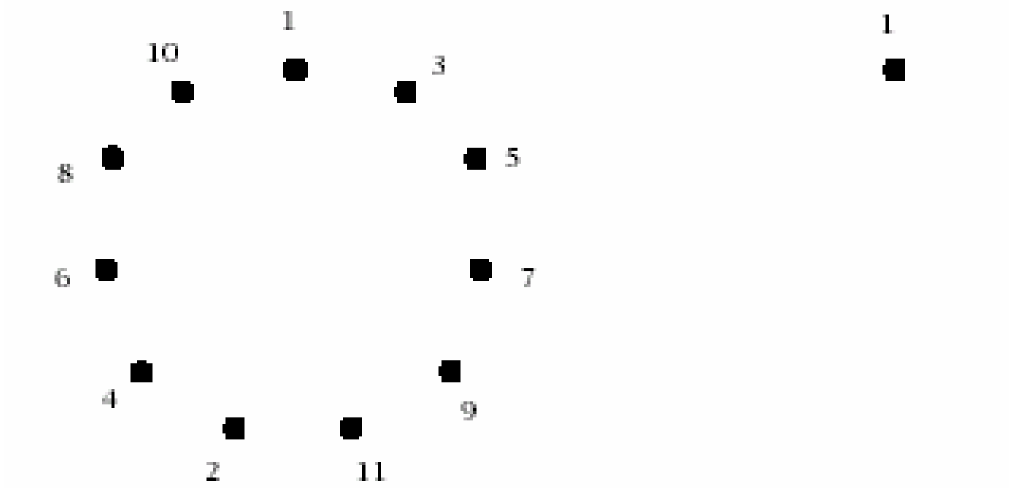


45. Construct a Nonagram and an 11-gram ❁ ❁

1. Use a finger to first trace the pattern, then use a pencil to draw it freehand.
2. Repeat the drawing in the space to the right by copying it freehand with a pencil.

   Notice the odd numbers start at the top and move clockwise down to the right, and the even numbers start at the bottom and move clockwise up to the left.



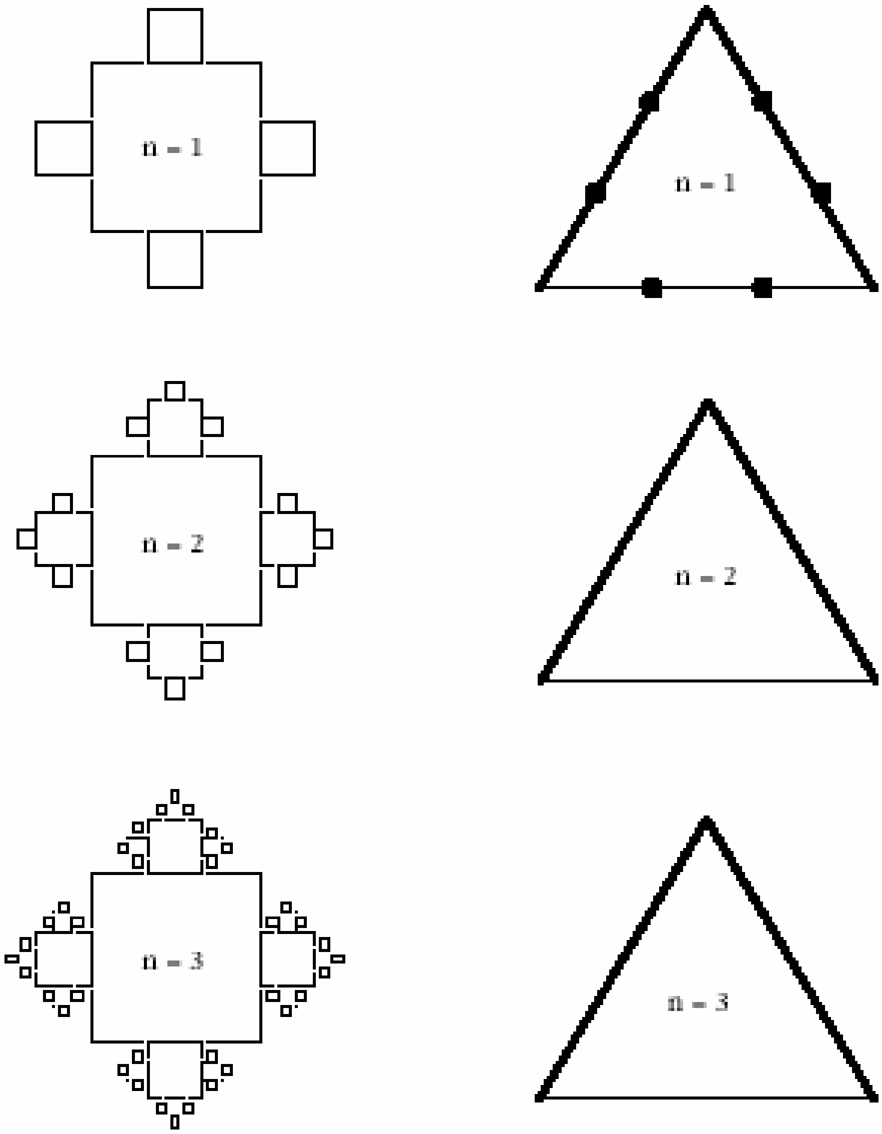


46. Square and Triangle Fractals

A fractal is a self-repeating pattern that recreates itself geometrically.

Below are simple 1-dimensional fractals around 2-dimensional polyhedra.

Put triangles on the sides of the triangle just like the squares on the squares.



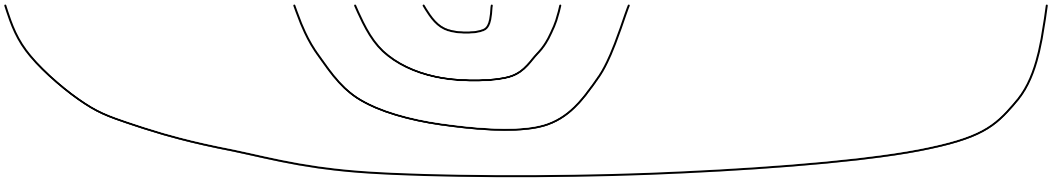
Chapter 7

Algebra

47. Gauss' Addition

Add up all the numbers from 1 to 100 the quick way with Gauss' addition.

1 + 2 + 3 + 4 + … + 48 + 49 + 50 + 51 + 52 + 53 + … + 96 + 97 + 98 + 99 + 100



Group the numbers in pairs, like the:

first (1) and the last (100) = ____1____ + __100__ = _______

second (2) and the second last (99) = ____2____ + __99__ = _______

third (3) and the third last (98) = _________ + ______ = _______

until the middle is reached

fiftieth (50) and the fiftieth last (51) = _________ + ______ = _______

So now there are ______ pairs of numbers, all adding up to __________.

This means the sum of 1 to 100 is _______ x _______ = _________ .

Add up all of the numbers from 1 to 1000 the quick way, with Gauss' addition.

1 + 2 + 3 +…+ 498 + 499 + 500 + 501 + 502 + 503 +…+ 997 + 998 + 999 + 1000

Group the numbers in pairs, drawing the numbers above together, like the:

first (1) and the last (1000) = _________ + _______ = _______

second (2) and the second last (999) = _________ + _______ = _______

third (3) and the third last (998) = _________ + _______ = _______

until the middle is reached

500th (500) and the 500th last (501) = _________ + _______ = _______

So now there are ______ pairs of numbers, all adding up to _________.

This means the sum of 1 to 1000 is ______ x _______ = _________ .

48. Running Totals ❀ ❀

The way to add a running total of a series of numbers is to add the units first, which are the right most digits, then the tens (1 digit in from the right), and then the hundreds, then the thousands, and so on. The formula looks like this: $h_1 t_1 u_1 + h_2 t_2 u_2 = h_1{+}h_2\ t_1{+}t_2\ u_1{+}u_2$,

If the addition is ten or more, write downthe units of the answer and carry the 1of the 10 leftwards and include it in the next sum.

For example, 1234 + 2345 becomes:

Do the units:	1234 + 2345		=	9
units	↑ ↑	4 + 5	=	9
Then do the tens:	1234 + 2345		=	79
tens	↑ ↑	3 + 4	=	7
Then do the hundreds:	1234 + 2345		=	579
hundreds	↑ ↑	2 + 3	=	5
Then do the thousands:	1234 + 2345		=	3579
thousands	↑ ↑	1 + 2	=	3

So the answer of 1234 + 2345 = 3579.

Calculate the sum of 4321 + 1234 using the running total method.

Do the units:	4321 + 1234		=	_________
units	↑ ↑	___ + ___	=	_________
Then do the tens:	4321 + 1234		=	_________
tens	↑ ↑	___ + ___	=	_________
Then do the hundreds:	4321 + 1234		=	_________
hundreds	↑ ↑	___ + ___	=	_________
Then do the thousands:	4321 + 1234		=	_________
thousands	↑ ↑	___ + ___	=	_________

So the answer of 4321 + 1234 = ______.

49. Trachenberg's Multiplication ❀ ❀ ❀

This is a simple method that allows multiplication without using pen and paper. Consider two 2-digit numbers, ab x cd, a and c are tens, b and d are units.

a x d (outside)
plus

a b x c d = a x c b x c (inside) b x d

tens units hundreds tens units

1. To calculate 12 x 34, consider a = 1, b = 2, c = 3, d = 4.
    - units are the right-most digits of both numbers = b x d = 2 x 4 = 8. Answer =___ ___ 8.
    - tens are the inside plus the outside multiples = (b x c) + (a x d) = 2 x 3 + 1 x 4 = 6 + 4 = 10. Here, the last digit for the tens is 0 and carry the 1 to hundreds. Abswer = (+1) _0_ _8_
    - hundreds are the left-most digits of both numbers = a x c = 1 x 3 = 3, plus 1 (carry) = 4. Answer = (3+1) 08 = 408.

So for 12 x 34, the answer in hundreds, tens and units = 408.

2. Calculate 23 x 15, consider a = ___, b = ___, c = ___, d = _______.
    - units are the right-most digits of both numbers = b x d = ___ x ___ = ___, carry = ___. Answer = ___ ___ ___ .
    - tens are the inside plus the outside multiples = (b x c) + (a x d)

      = (___ x ___) + (___ x ___) = _____ + _____ + carry ___ = ___, carry = ___. Answer = ___ ___ ___ .
    - hundreds are the left-most digits of both numbers = a x c

      = ___ x ___ + carry ___ = _____. Answer = ___ ___ ___ .

So for 23 x 15, the answer in hundreds, tens and units = _________.

3. Calculate 14 x 24, consider a = ___, b = ___, c = ___, d = ___.
    - units are the right-most digits of both numbers = b x d = ___ x ___ = ___, carry = ___. Answer = ___ ___ ___ .
    - tens are the inside plus the outside multiples = (b x c) + (a x d)

      = (___ x ___) + (___ x ___) = ___ + ___ + carry ___ = ___, carry = ___. Answer = ___ ___ ___ .
    - hundreds are the left-most digits of both numbers = a x c

      = ___ x _____ + carry ___ = ___. Answer = ___ ___ ___ .

So for 14 x 24, the answer in hundreds, tens and units = _________.

50. Single Variable Equations

Calculate the value of χ by working backwards and rearranging the equations.

1. Addition

χ	+	4	=	6,	χ = 6 - 4 = ______
22	+	χ	=	47,	χ = ______ - ______ = ______
101	+	χ	=	201,	χ = ______ - ______ = ______
χ	+	9999	=	10101,	χ = ______ - ______ = ______

2. Subtraction

χ	-	4	=	6,	χ = 6 + 4 = ______
χ	-	10	=	55,	χ = ______ + ______ = ______
200	-	χ	=	100,	χ= ______ - ______ = ______
8888	–	χ	=	1111,	χ = ______ - ______ = ______

3. Multiplication

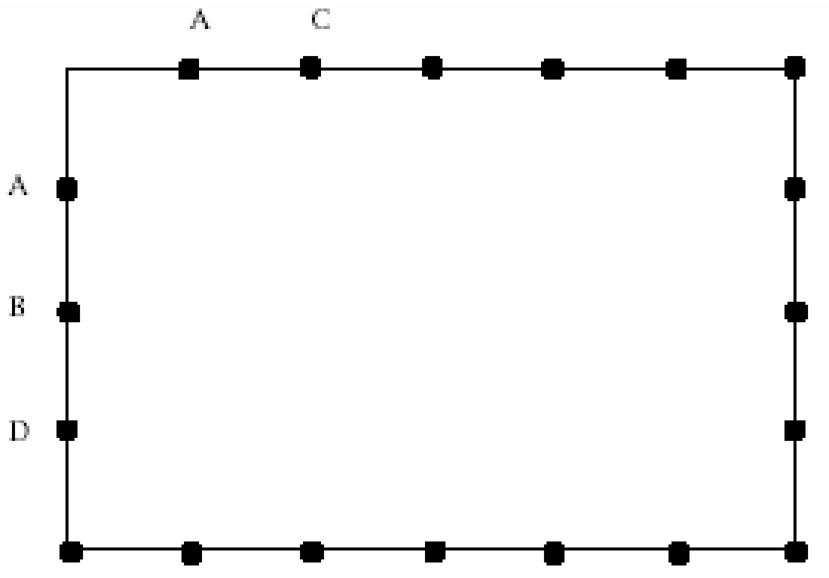
5	x	χ	=	20,	χ = 20 ÷ 5 = ______
15	x	χ	=	120,	χ = ______ ÷ ______ = ______
χ	x	256	=	64,	χ = ______ ÷ ______ = ______
100	x	χ	=	5,	χ = ______ ÷ ______ = ______

4. Division

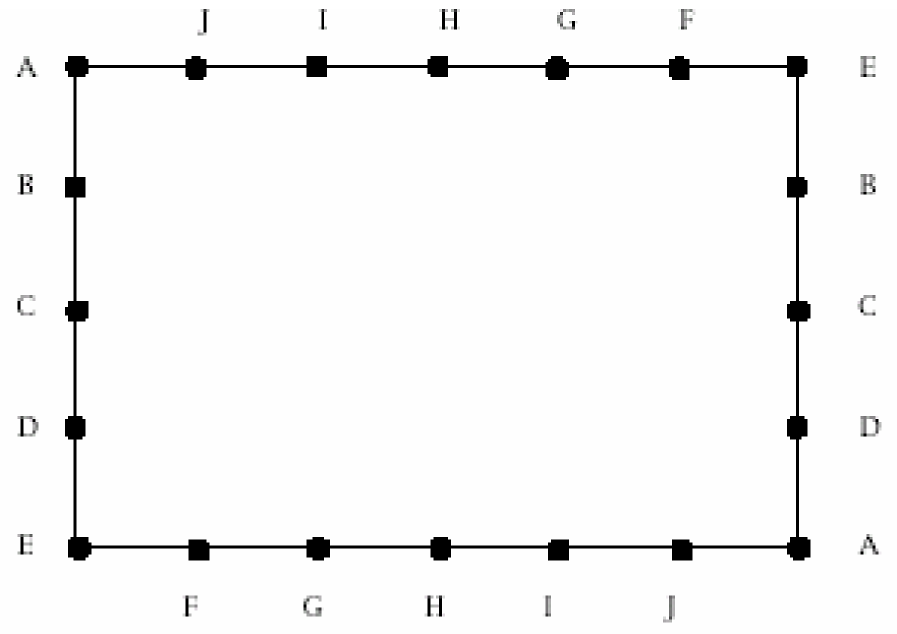
χ	÷	8	=	2,	χ = 2 x 8 = ______
χ	÷	10	=	500,	χ = ______ x ______ = ______
999	÷	χ	=	99.9,	χ = ______ ÷ ______ = ______
22	÷	χ	=	3.14,	χ = ______ ÷ ______ = ______

51. Three and Four Colour Map Painting

1. Use a pencil to connect AA, then from the middle of that curve AA to point B. Draw a curve from the middle of the last curve to point C and the middle of that curve to point D.
2. Continue drawing a series of curves that always connect in a 3-way junction. Draw from the middle of one curve to another, and from the curves to dots.



3. Use a pencil to connect the letters AA, BB, CC, and so on, with gentle curvy lines, so that they cross over each other creatively with big even spaces.



4. Use only 4 colours to fill in the sections so no two colours share a boundary.

52. Linear Solutions

This exercise describes 10 equations (down) of 10 variables each (across). By colouring in squares you are solving one variable for each of the equations.

For example, a line may be described as: $a + b + c + d + e + f + g + h + i + j = 0$

1. Use a pencil to colour in as many squares as you can without having two or more in the same row or column. That's only one for every straight line across or down.

   There are ____ coloured square single linear solutions in this 10 x 10 matrix.

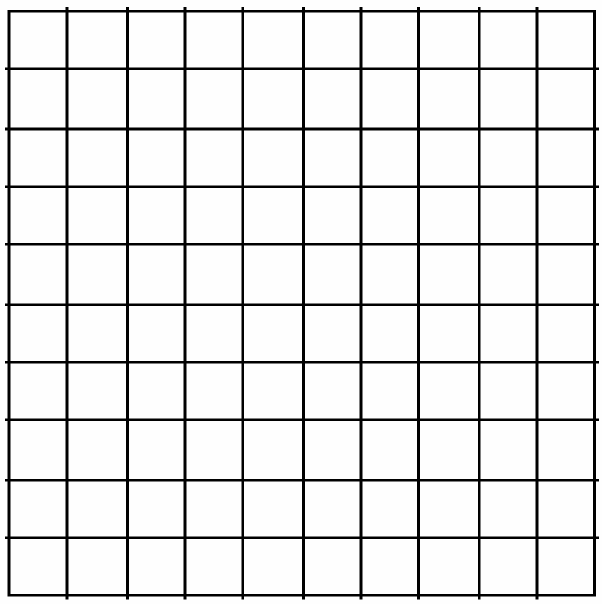
2. Use a different coloured pencil to colour in as many squares as you can without having three in a row. That is, only two, one of each colour, in a straight line.

   There are ____ coloured square double linear solutions in this 10 x 10 matrix.

3. Use a different coloured pencil to colour in as many squares as you can without having four in a row. That is, only three, one of each colour, in a straight line.

   There are ____ coloured square triple linear solutions in an 10 x 10 matrix.

4. Continue using different coloured pencils to fill in one linear solution at a time.



53. Connectivity – Part 1 (6 points) ❀ ❀

1. Use a ruler and coloured pens to join lines from these points to the other points.

   Red: 1-2, 1-3, 1-4, 1-5, 1-6.
   Green: 2-3, 2-4, 2-5, 2-6.
   Blue: 3-4, 3-5, 3-6.
   Black: 4-5, 4-6.
   Any: 5-6.

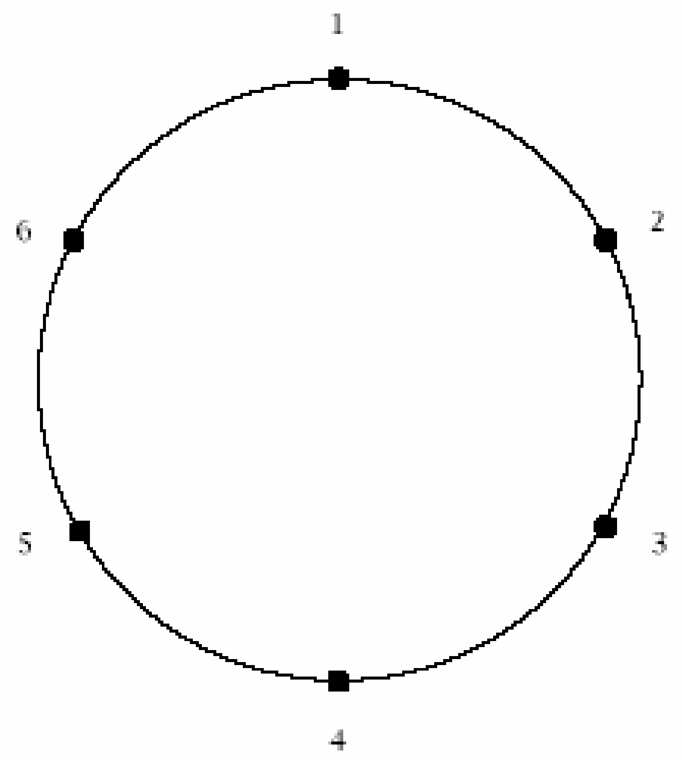
   There are ___ lines in total.

2. Calculate how many lines connect the 6 points with the formula for (n) points.

   Connectivity = $(n^2 - n) \div 2$, where n = number of points

   where n = 6, = $(6^2 - 6) \div 2 = (6 \times 6 - 6) \div 2$

   [calculator] 6 x 6 = - 6 = ÷ 2 = ____ lines connecting 6 points.



54. Connectivity – Part 2 (12 points) ❀ ❀

1. Use a ruler and coloured pens to join lines from these points to the other points.

Red: 1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8, 1-9, 1-10, 1-11, 1-12.
Green: 2-3, 2-4, 2-5, 2-6, 2-7, 2-8, 2-9, 2-10, 2-11, 2-12.
Blue: 3-4, 3-5, 3-6, 3-7, 3-8, 3-9, 3-10, 3-11, 3-12.
Black: 4-5, 4-6, 4-7, 4-8, 4-9, 4-10, 4-11, 4-12.
Red: 5-6, 5-7, 5-8, 5-9, 5-10, 5-11, 5-12.
Green: 6-7, 6-8, 6-9, 6-10, 6-11, 6-12.
Blue: 7-8, 7-9, 7-10, 7-11, 7-12.
Black: 8-9, 8-10, 8-11, 8-12.
Red: 9-10, 9-11, 9-12.
Green: 10-11, 10-12.
Blue: 11-12.

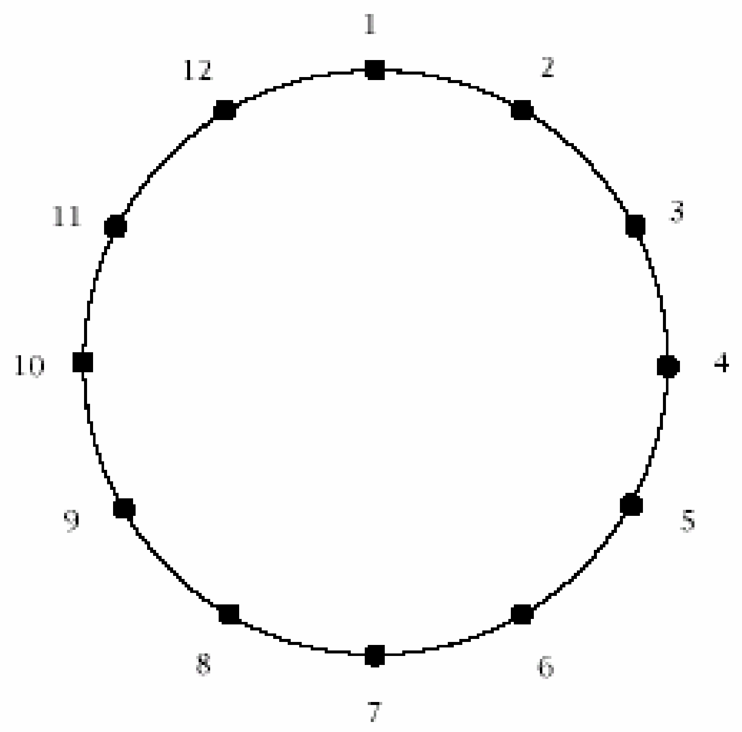
There are ___ lines in total.

2. Calculate how many lines connect the 12 points with the formula for (n) points.

Connectivity = $(n^2 - n) \div 2$, where n = number of points

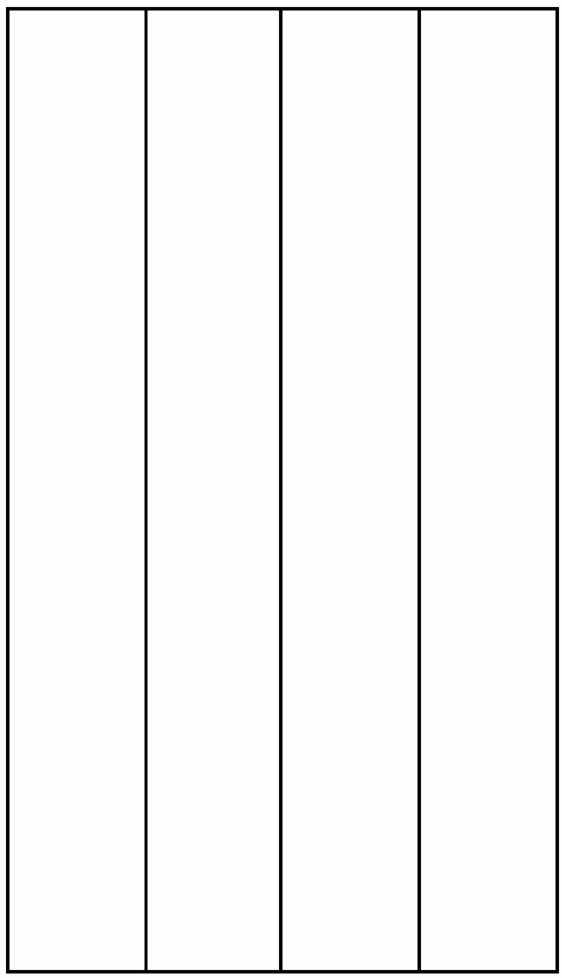
where n = 12, = (____ ^{2} - ____) ÷ 2 = (____ x ____ - ____) ÷ 2

[calculator] ____ x ____ = -______ = ______ ÷ 2 = ____ lines.



55. Rings – Part 1 (Splitting into Two)

1. Get an A4 sheet of paper and fold it down the middle lengthways, creating two long, thin panels. Fold it again to create 4 thin strips.
2. Use scissors to cut along the creases to create 4 long thin strips of paper.
3. Take 1 strip and use sticky tape to tape the ends together by overlapping the paper by about 1 cm and taping the width of the paper on both sides.
4. Use scissors to cut the paper down the middle of the strip lengthways, going around the ring in a circle, creating two separate even thinner rings.



56. Rings – Part 2 (The Mobius Strip) ❁

1. Take another of the four thin strips of paper and hold the ends together, and holding one end still, twist the other just once, flipping it over, before taping it together.
2. Use sticky tape to tape the ends together as before, by overlapping the paper by about 1 cm and taping the width of the paper on both sides.

   This is called a Mobius Strip and only has 1 side. You can see two edges and two sides, but mathematically both sides are the same side.
3. Use a pencil to trace along the path of one side and keep going until you get back to where you started, proving the pencil never left the paper and thus the Mobius Strip only has 1 side.
4. Use scissors to cut the paper down the middle of the strip lengthways, going around the ring in a circle, following the pencil line.

   How many sides has this ring got? ____________________

   How many twists has this ring got? ____________________

   How big is it? ____________________

57. Rings – Part 3 (The Mobius Rings) ❁

1. Create a double length strip of paper by taping the 3rd and 4th strips together, overlapping them by 1cm and taping both sides.
2. Put the ends of the double length strip together and then twist one end just once, flipping it over. Then twist the same end again, flipping it over for the second time, putting a total of two twists in the strip.
3. Use sticky tape to tape the ends together as before by overlapping the paper by about 1 cm and taping the width of the paper on both sides.
4. Use scissors to cut the paper down the middle of the strip lengthways, going around the ring in a circle.

   How many rings do you have? ____________________

   What is special about them? ____________________

58. Cartesian Mapping ❁

Cartesian coordinates are named after the French mathematician and philosopher Descartes, and are given in the formula:
(first number = across, , second number = up).

The first number informs how many column places across, from left to right.

The second number informs how many row places up, from bottom to top.

For example (6,3) means 6 places right and then 3 places up from (0,0).
0.

1. Use a pencil to plot the following Cartesian coordinates on the number grid on the next page.
2. Label each point with its letter next to it, using very neat printed capital letters.
3. Use a pencil to draw the curved line of best fit to join the dots and draw the map. This is a map of __________________.

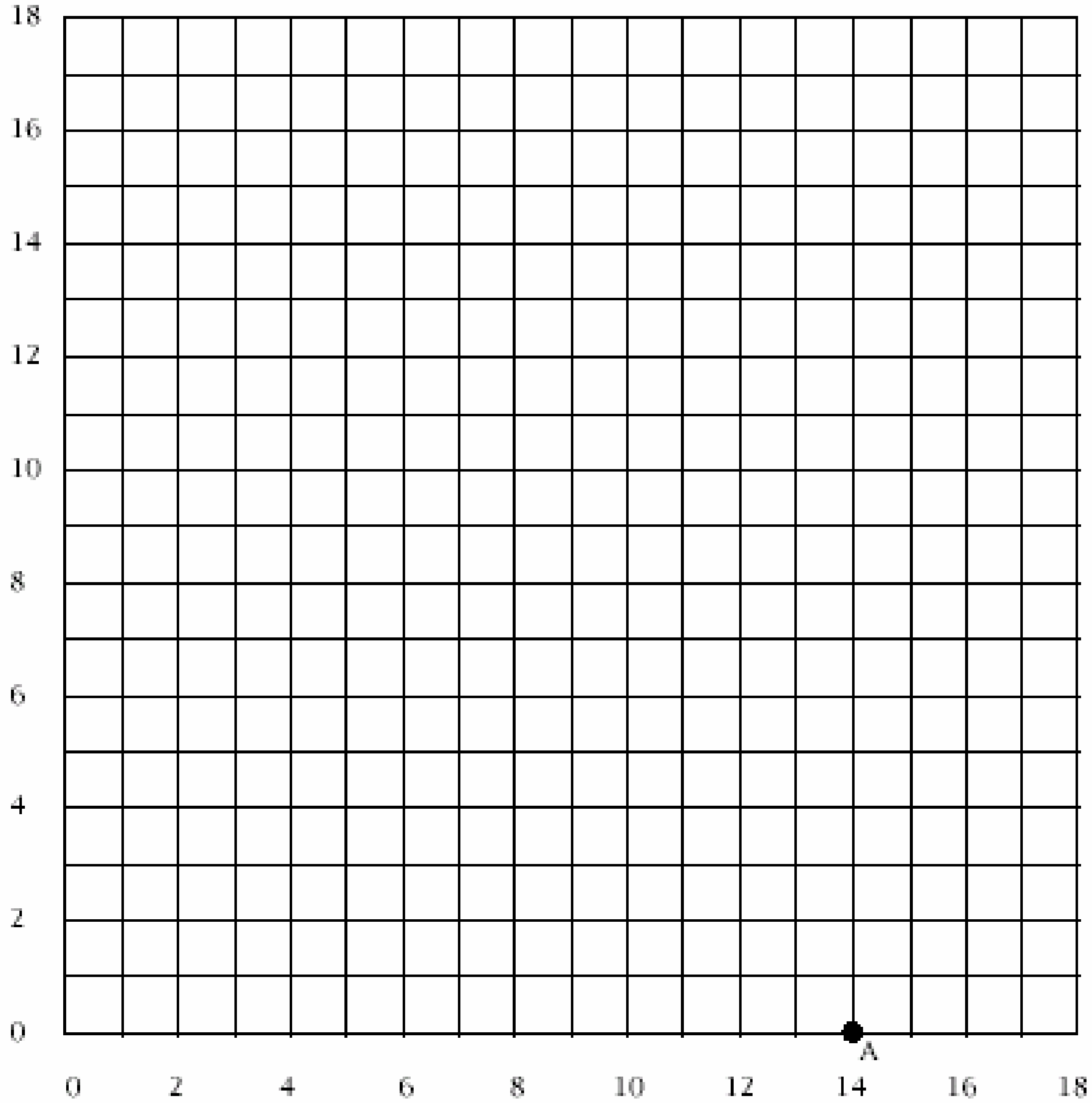
A (14, 0)
B (13, 2)
C (15, 2)
D (14, 3)
E (16, 4)

F (18, 8)
G (16, 13)
H (15, 16)
I (14, 18)
J (12, 13)

K (10, 16)
L (10, 17)
M (8, 17)
N (7, 16)
O (5, 16)

P (4, 14)
Q (0, 10)
R (1, 7)
S (3, 5)
T (5, 6)

U (8, 7)
V (9, 6)
W (10, 5)
X (11, 6)
Y (11, 5)
Z (12, 4)


18
16
14
12
10
8
6
4
2
0
A
0
2
4
6
8
10
12
14
16
18

59. Prime Number Formula ❁ ❁

A prime number is a number that can only be divided by itself and the number 1. The word prime comes from the Latin word primus, which means 'first'.

You can find all the primes (the lonely numbers) under 100 by using a pencil and ruler:

1. Remove all the products of 2 (and 4 and 6 and 8 and 10 and so on):
   - Put a line through all of the numbers that are divisible by 2 (the even numbers), except 2:
   - Use a ruler to draw a diagonal lines through 10, 28-4, 46-6, 64-8, 82-18, 92-36, and so on.
2. Remove all the products of 3 (and 6 and 9 and so on):
   - Put a line through the three columns that add up to 3, 6 and 9: Use a ruler to draw vertical lines through 3-93, 6-96, 9-99.
3. Remove all the products of 5 (and 10 and 15 and 20 and so on):
   - Put a line through all of the number that are divisible by 5, not including 5:
   - Use a ruler to draw a diagonal lines though 15-45, 55-95, 10-90.
4. Remove all the products of 7 still remaining:
   - Put a small dash through the numbers 49 and 77.
5. Go down columns 1, 2, 4, 5, 7, 8 and circle all of the remaining numbers.

These are the first 26 prime numbers and only have 1 and themselves as factors.

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81
82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99

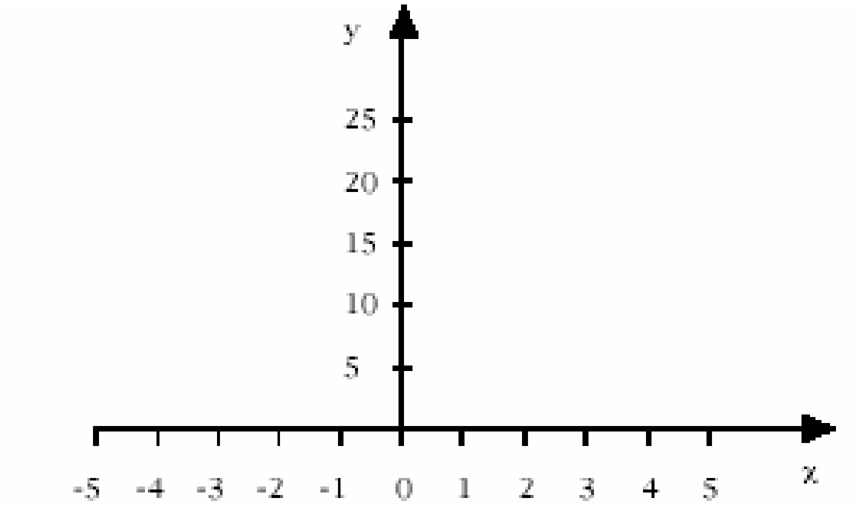
60. Polynomial Functions

A polynomial function is a formula where the letters represent a moving or changing number. For example, y = 5.t, means when t=1, y= 5 x 1 = 5.

Use a calculator to fill in the following table and graph the coordinates generated.

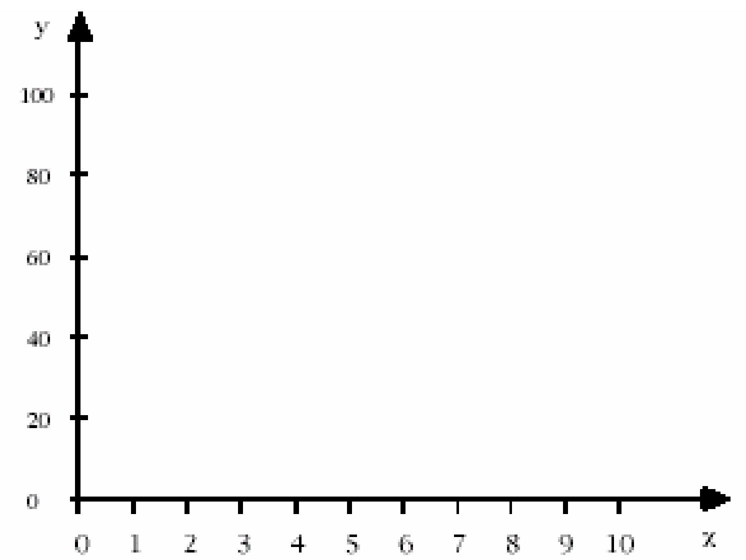
Function 1: $y = \chi^2$ for the range χ = -5 to χ = 5.
For example y = -5 x -5 = 25 (for χ = -5)

χ	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	25										



Function 2: $y = \chi^2 - \chi$ for the range χ = 0 to χ = 10.
For example y = 3 x 3 - 3 = 6 (for χ = 3).

χ	0	1	2	3	4	5	6	7	8	9	10
y				6							



About the Author

Jason Betts is mathematician and philosopher living in Tasmania.

Dr Betts has a Bachelor of Science Degree in Mathematics from the University of Tasmania, a postgraduate Diploma in Metaphysical Science from Lindlahr College, WA and an honorary Doctor of Science Degree and a Doctor of Philosophy from Open International University of Complementary Medicines, Sri Lanka.

Dr Betts enjoys teaching mathematics and philosophy privately and in public high schools. His personal interests are in historical mathematics and its influence on philosophical thought, especially in the ancient western mystery school tradition, such as the Pythagorian and Platonic Academies.

This amazing book is his contribution to the children (and adults) of today and to the thinkers of tomorrow, unveiling simple and visual patterns in a process of selfdiscovery of self-evident mathematical truths.

Dr Betts constantly struggled with high school maths – borderline passes, borderline failures, headaches and agony, but he needed it for science. Then, amazingly, he was taught to 'see the pattern', and maths suddenly became more fun, exciting and less of a burden and more of a puzzle of knowledge.

In teaching a child to see the patterns of maths at an early age, maths becomes less of a confusing subject of rules and numbers, and more of a subject of patterns, meaning and understanding. This is the book that Jason wanted when he was failing high school maths. Any child can have it now.